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Chief Editor
Dr. S.K. Arora
Managing Editor
Dr. D.K. Diwan
Co-Editor
S.B. Nangia

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# भारतीय लोकतन्त्र : चुनौतियाँ व समाधान

डॉ० ममता रानी*

भारत दुनिया का सबसे बड़ा लोकतांत्रिक राज्य माना जाता है जिसकी आधारशिला लोकतान्त्रिक मूल्यों के आधार पर रखी गई है और इस विशाल लोकतन्त्र में अनेकों विचारधाराओं का समावेश किया गया है। भारतीय संविधान की प्रस्तावना भारत को सम्पूर्ण प्रभुत्व सम्पन्न, समाजवादी, धर्मनिरपेक्ष, लोकतान्त्रिक गणराज्य घोषित करती है जिसमें गूढ रूप से जन कल्याण की अवधारणा के सरोकार देखने को मिलते है। अमेरिकी पूर्व राष्ट्रपति अब्राहिम लिंकन के अनुसार, लोकतन्त्र जनता का, जनता के लिए और जनता के द्वारा शासन है। उनकी यह यह परिभाषा प्रामाणिक मानी जाती है। लोकतन्त्र में अन्तिम सत्ता जनता के हाथों में होती है और उसी के अनुसार शासन व्यवस्था को चलाया जाता है। लोकतन्त्र का उद्देश्य जनता का कल्याण करना है।

लोकतन्त्र मात्र एक शासन प्रणाली ही नहीं अपितु एक विशिष्ट प्रकार के राजनीतिक संगठन, सामाजिक संगठन, आर्थिक व्यवस्था तथा एक नैतिक और मानसिक भावना का भी नाम है। लोकतन्त्र मानव जीवन का वह समग्र रूप है जिसकी व्यापक परिधि में उसके जीवन से सम्बन्धित सभी पहलु आ जाते हैं। हालांकि लोकतन्त्र का रूप शताब्दियों से परिवर्तनशील रहा है परंतु स्वतन्त्रता, समानता, न्याय व भातृत्व के सिद्धान्त सदैव इसके आधार स्तम्भ रहे है।

भारतीय लोकतन्त्रात्मक शासन व्यवस्था जन सहमति पर आधारित एक आदर्श व्यवस्था का स्वरूप है। लोकतन्त्र केवल एक शासन स्वरूप नहीं अपितु एक सामाजिक संगठन भी है। सामाजिक आदर्श के स्वरूप में लोकतन्त्र वह व्यवस्था है जिसमें धर्म, जाति, लिंग, जन्म, स्थान आदि के आधार पर कोई भेदभाव नहीं किया जाता। सही अर्थो में लोकतन्त्रीय समाज ही लोकतन्त्रीय राज्य का आधार हो सकता है।

हम भारतीय सौभाग्यशाली रहे है जिन्हें बिना किसी आन्दोलन और क्रान्ति के एक लोकतान्त्रिक व लिखित संविधन प्राप्त हुआ है और वो तमाम अधिकार हमें प्रदान किए गए है जिनको पाने के लिए इंग्लैण्ड, अमेरिका व फ्रॉस जैसे देशों में क्रान्तियॉं हुई हैं। परन्तु जिस लोकतान्त्रिक व्यवस्था की कल्पना हमारे संविधान निर्माताओं ने की थी, वास्तविकता के धरातल पर यदि देखा जाए तो भारतीय लोकतन्त्र उन सभी आदर्श सिद्धान्तों से कोसों दूर नजर आता है। स्वतन्त्रता के इतने समय बाद भी ये प्रतीत होता है कि हमारे संविधान निर्माताओं ने इस आदर्श संविधान के माध्यम से जिन आदर्शो को पाने का साध्य निश्चित किया था, हमारा देश उसकी विपरीत दिशा की ओर अग्रसर होता जा रहा है परिणामस्वरूप साध्य तक पहुँचने का रास्ता और लम्बा होता जा रहा है।

सुभाष काश्यप ने अपनी पुस्तक, 'भारतीय राजनीति और संविधान : विकास, विवाद और निदान' में बहुत ही सार्थक शब्दों में लिखा कि लोकतन्त्र की भूमिका का मूल्यांकन करने का एक उपाय हो सकता है कि देखा जाए कि हमारे संविधान निर्माताओं और राष्ट्रनायकों ने किन उद्देश्यों, आशा–आकांक्षाओं को लेकर वर्तमान राजनीतिक व्यवस्था बनाई थी और गत वर्षो में हम उन्हें मूर्त रूप देने में कहाँ तक सक्षम सफल सिद्ध हुए है।

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संविधान निर्माताओं ने कभी सोचा भी नहीं होगा कि उनके सपनों के लोकतन्त्र को आने वाले समय में इतनी बुराइयाँ जकड लेगी। आज भारतीय लोकतन्त्र के मार्ग में अनेको समस्याएँ और चुनौतियाँ आन खडी हुई है। आतंकवाद, भ्रष्टाचार, आर्थिक विषमताएँ, जातिवाद, सत्ता लोलुपता, सामाजिक हिंसा, क्षेत्रीय असन्तुलन, राजनीति मे भाई–भतीजावाद, वंशवाद, अपराधीकरण, न्यायिक प्रक्रिया में देरी, कालाबाजारी, आदि समस्याओं ने लोकतन्त्र को खोखला कर दिया है।

किशन पटनायक ने अपनी रचना, 'भारतीय राजनीति पर एक दृष्टि – गतिरोध, सम्भावना और चुनौतियाँ' मे माना है कि दुनिया के सबसे बडे लोकतान्त्र मे दुनिया के सबसे ज्यादा गरीब, कुपोषित और अशिक्षित लोग रहते है। इस मायने में लोकतन्त्र को आधी से अधिक सदी गुजर जाने के बाद भी करोडों मनुष्य कगाली और फटेहाली मे जीते है। अनेकों विषमताएँ बढी व मजबूत हुई है। ये खाइयाँ और दरारें इतनी अधिक बढ गई है कि सम्पूर्ण राष्ट्र बिखरता नजर आ रहा है। घूसखोरी से लेकर महाघोटालों तक भ्रष्टाचार अपनी चरम सीमा पर है। भारत की राजनीति यहाँ के राजनीति दल अनैतिकता, सिद्धान्तहीनता और अवसरवाद के मूर्त रूप मे नजर आते है। वहीं सुरेन्द्र मोहन ने 'वर्तमान राजनीति की ज्वलन्त चुनौतियाँ' नामक पुस्तक में लिखा है कि राजनीतिक दलो में लोकतन्त्र के प्रति न तो कोई निष्ठा शेष है और न ही उनके निराश टिकटार्थी अनुशासन मे बंधे रहते है। लोकतन्त्र मे सबको अपने मान व आस्थाएँ बदलने का पूरा अधिकार है किंतु विवेक, तर्क या जनहित के दबाव के चलते। अब जब मात्र पद, संसद सदस्यता और सत्ता ही इन्हें बदलने की मुख्य प्रेरणाएँ बन जाए और टिकट हाट में निलाम होने लगे तो लोकतन्त्र की जडे हिलने लगती है। प्रलोभन जब बडे–बडों को फुसला दे तो मुल्यों व नीतियों में आस्थाओं वाली राजनीति की अर्थी ही निकल गई समझिए।

लोकतान्त्रिक व्यवस्था से आमजन का मोह भंग होता जा रहा है। भ्रष्टाचार रूपी वृक्ष की जडे इतनी फैल गई है जिसे उखाड पाना असम्भव सा प्रतीत होता है। न्याय व्यवस्था तो समाज की साधारण आशाओं पर भी खरी नही उतर पाई है। लोगों को न्याय या तो मिलता ही नहीं और यदि मिलता भी है तो अत्यधिक देरी से, जिसका कोई औचित्य ही नहीं है और उनका खर्च भी अन्धाधुँध हो जाता है एंव इस धीमी न्यायिक प्रक्रिया में भी अनेको दाँव पेच है। हमारी स्वतंत्र व निष्पक्ष न्यायिक व्यवस्था भी कार्यपालिका व विधानपालिका के प्रभाव से स्वय को मुक्त कर पाने में सफल नहीं हो सकी है। राजनीति का अपराधीकरण व अपराध के राजनीतिकरण के फलस्वरूप बाहुबल, धनबल और माफिया शक्तियों के प्रभाव मे अत्यन्त इजाफा हुआ है। जातिवाद व सम्प्रदाय के इस खण्डित लोकतन्त्र में राष्ट्रीय एकता व अखण्डता मात्र राजनीतिक मंचो के नारे ही सिद्ध हो पाई है।

सुभाष काश्यप लिखते है कि राष्ट्र निर्माण का आधार है राष्ट्रीय एकता और मूलभूत प्रश्नों पर मतैक्य। जबकि सत्ता के सौदागरों के लिए सबसे अधिक मूल्यवान है वोटो क गणित, वह तो पलते–पनपते है फूट पर, विखण्डन विभाजन पर, बंटवारे पर। जाति, उपजाति, भाषा, प्रदेश के आधार पर समाज के टुकडों में बाटकर, एक–दूसरे के विरूद्ध लडाकर, अपने–अपने समूह की अलग पहचान बनाने में निहित स्वार्थ पैदा कर चुनावी पैतरेबाजी करना और अपना उल्लु सीधा करना ही इस व्यवस्था के राजनैतिक दलो का काम है। यही कारण है कि अंग्रेजी ढंग से लोकतन्त्र के अन्तर्गत राष्ट्रीय एकता का, एक राष्ट्रीय भावना के निर्माण का, एक संगठित भारतीय समाज की पहचान बनाने का हमारा उद्देश्य पूरा न हो सका।

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किशन पटनायक के अनुसार आधुनिक विकास पद्धति क्षेत्रीय विषमता, आन्तरिक उपनिवेशवाद, राष्ट्रीयता की सतही व संकीर्ण अवधारणा, सत्ता का अत्यधिक केन्द्रीयकरण, सत्ता की ओछी राजनीति आदि भारतीय लोकतन्त्र की सामूहिक विफलता है।

यहाँ यह कहना ठीक नहीं होगा कि इन चुनौतियों से निजात नहीं मिल सकती आवश्यकता है तो महज एक सामूहिक प्रयास की। यदि हम राष्ट्रीय एकता व अखण्डता के लिए एक सामूहिक कोशिश करें तो इन समस्याओं से निजात पा सकेंगे। आवश्यकता है भेड़ चाल से अलग होकर कुछ क्रान्तिकारी कदम उठाए जाएं। समस्याओं व चुनौतियों की व्याख्या संकीर्ण हितों से ऊपर उठकर तार्किक आधारों पर की जाए। भ्रष्टाचार का मुख्य कारण निर्वाचन प्रक्रिया व राजनेताओं की नियत है। निर्वाचन पर इतना अधिक खर्च होता है कि जिसका हिसाब लगा पाना नामुमकिन है इसी के चलते अनैतिक प्राथाओं का विकास होता है। धन की बढ़ती चमक ने लोकतन्त्र की सूरत को धूमिल कर दिया है। समूचे शासन में पारदर्शिता लानी होगी।

भारतीय लोकतन्त्र के समक्ष चारित्रिक संकट है और यह व्यक्तिगत भी और सामूहिक भी। जब तक जीवन का अन्तिम लक्ष्य धन होगा तब तक भ्रष्टाचार खत्म नहीं हो सकता। ऐसे राजनेताओं की तलाश करनी होगी जो राजनीति को पेशा न मानकर समाज सेवा एंव राष्ट्र सेवा का अवसर समझे। अभिभावकों व शिक्षक का उद्देश्य बच्चों का चरित्र निर्माण करना होना चाहिए ताकि राष्ट्र को जानदार व शानदार बनाने में भूमिका निभा सकें।

देश के प्रत्येक नागरिक को संकीर्ण भावनाओं से ऊपर उठकर राष्ट्रभक्ति करनी होगी ताकि भ्रष्ट राजनेता अपने मनसूबों में कामयाब न हो सके। पुलिस व न्याय व्यवस्था की खामियों को दूर करना होगा देश के युवा को अनैतिक विचारों को त्यागते हुए चरित्रवान बनना होगा।

हमारे देश में सबसे बडी समस्या यही है कि राजनैतिक दल किसी भी विचार को लेकर एकमत नहीं होते है यहाँ तक राष्ट्रहित जैसे विचार पर भी नहीं। राष्ट्रहित जैसे विचारों पर राजनैतिक दलों को एकजुट होकर राष्ट्रीय मनोबल को बढ़ाना होगा। हमें कभी भी यह नहीं भूलना चाहिए कि अपने आदर्शों से भटकने वाले राष्ट्र नष्ट हो जाते है। हर देशभक्त विशेषतौर पर युवाओं को भ्रष्टाचार व अनैतिकता के खिलाफ आगे आने चाहिए। सामाजिक न्याय के साथ–साथ सामाजिक आर्थिक विषमताओं के खिलाफ भी एकजुट होना पडेगा। लोकतन्त्र की सुरक्षा हेतु इसके आर्थिक आधार को परिभाषित करना होगा। डॉ0 बी0 आर0 अम्बेडकर ने जिस आर्थिक लोकतान्त्र की दलील दी थी, वही लोकतन्त्र के अस्तित्व की गारण्टी है उसी से सामाजिक समानता की शर्त पूरी होती है। लोकतन्त्र की सफलता के लिए जागरूकता व शिक्षा का होना अत्यन्त आवश्यक है। इसके साथ ही न्यायपालिका, नौकरशाही व प्रैस को भी लोकतन्त्र की सफलता में सतर्क भूमिका निभानी होगी।

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# Social Sustainability in India: Challenges and Hurdles

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*Abstract:* Sustainable development is a process and organizing principle for sustaining finite resources necessary to provide for the requirement of the next generation for life on the planet. As defined by the United Nations, Sustainable development is a common agenda for global concern which everybody agrees upon, but bringing this global concern into public policies is a difficult task.1 Social development is a method of using of resources in a manner that aims to satisfy human needs while maintaining the environment so that these needs can be fulfill not in the present time but also for the future generation. Sustainable development focuses upon a relationship between humans and their environment and indicates a warning that humans cannot push development, which is against nature as in the end it is always the nature, which is going to win. If sustainable development is to be successful, the attitudes of individuals as well as governments with regard to our current lifestyles and the impact they have on the environment will need to be changed. Sustainable development has some forward looking and broad based objectives, which transcend class, caste, language and regional barriers.

*Keywords:* Sustainable development

## 1. Introduction

Sustainable development is based on social sustainability. In the present scenario of Indian society, it becomes more important for the study that how the social sustainability effects the sustainable development. This paper, 'Social Sustainability in India : Challenges and Hurdles' enquire the challenges and hurdles before the social sustainability to meet the objective of the sustainable development.For the proper perspective of the study it is necessary to define the social sustainability. Social sustainability is the ability of social system. According to the Western Australia Council of Social Services (WACOSS), Social sustainability occurs the formal and informal processes, systems, structures and relationships activity support the capacity of current and future generations to create healthy and livable communities and equitable. diverse, democratic and provide a good quality of life.2

Social sustainability is a process for creating sustainable, successful places that promote wellbeing by understanding what people need from the places they live and work. Social sustainability combines design of the physical realm with design of social world- infrastructure to support social and cultural life, social amenities, system for citizen engagement and space for people and place to involve.3

The three pillars of sustainability are the powerful tools for the defining the complete sustainability problem. It consists of at least economic, social and environmental pillars. 'I' any pillar is weak then the system as a whole in unsustainable.[4]

| Sustainability | | | | |
|---|---|---|---|---|
| Social | | Environment | | Economic |

Social Sustainability has three components 'Development' is concerned with meeting the basic requirements, 'Bridge Sustainability' focuses the changing behaviour and 'Maintenance Sustainability' refers to social acceptance or

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what can be sustained in social terms.[5] Social Sustainability is a value based development directed towards the improvement of quality of social life of the people. It has to be focused on removal of poverty, ignorance, discrimination, disease and unemployment.

The concept of "Social sustainability" encompasses the social equity, livable, community development, social capital, social support, human rights, labor right, social responsibility, social justice, cultural competence and human adaption.

## 2. Challenges and Hurdles for the Social Sustainability

There are so many challenges and hurdles in India for the social sustainability. Social justice and equality are the major problems for the social sustainability. Over the past few decades, the Indian Society has become more and more divided between rich and poor. This inequality encourage injustice in the society.

Social inequality assumes a particularly reprehensible form in relation to the backward classes and communities which are treated as untouchables and so the problem of social justice is an urgent and important in India. Here using the term Social Justice in a comprehensive sense so as to include economic justice. The concept of social justice thus takes within its sweep the objective of removing all inequalities and affording equal opportunities to all citizens in social affairs as well as economic activities.[6] Social and economic progress remains uneven. Rising inequalities and social environment degradation are the major problems before the social sustainability.

The concept of social justice has varied with age and time; social justice is to the millions "to wipe every tear for every eye"[7] Be that as it may, our constitution envisages tripartite picturesque of social justice that is justice-socio, economic and political, is secured by fundamental rights. Social sustainability developed when social justice is successful in society.

Women's increased participation in the paid work force has clearly been tied to the development of capitalism and a focus on individual workers earning a wage on which they could live, as opposed to more traditional family cooperation in accumulating the means to live. Women's participation in public as well private worlds is related to the growth in the belief in human rights including the doctrine of individual rights."[8]

In the present criminal system in India, the police, the judiciary and the prison authority, have generally failed to check organized crimes in the society. Basically the laws of the land tend to favor of accused. Jails are also hardly a deterrent to a criminal. The active members of the criminal gangs including their bases, money collectors, liaison worker and ancillary workers, and conspiratators operating inside the country while international connections usually escape the clutches of law for want of adequate provisions the criminal law.

Problems of child labour is basically due to the poverty of the people. Total eradication of child labour is not possible in a development economic like India. But child labour should not be encouraged on ethical and social considerations.

## 3. Conclusions

In Short, prevention and detection of inequality, social injustice and crime are the primary duty of the state for the social sustainability. There is need for change in the existing system and governance, policies and strategies of the state and as well as in the police department and judiciary for the purpose of social justice. Effective governance structure required to monitor and ensure the social sustainability. A framework has to be provided to decide developmental actions by the nation, communities and individuals. Balanced policies are required for the sustainability in society.

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# Exponential Strike-Slip Dislocation Model in Dissimilar Anisotropic Media

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## ABSTRACT

Mathematical dislocation model has the ability to compute the deformation field due to well defined arbitrarily shaped faults with arbitrary slip distributions. Explicit expressions for the deformations at any point of two homogeneous monoclinic elastic half spaces with different interface conditions due to variable slip along a very long strike slip dislocation situated in the lower elastic half space have been obtained. The interface may be perfect, stress free or rigidly clamped. The variation of the displacements with horizontal distance from the fault trace due to exponential slip profile is compared to study the effect of different types of interfaces between anisotropic elastic media. The effect of anisotropy on the displacement field is also examined. Numerically, it has been observed that the anisotropy has a significant effect on the displacement and also the field is significantly influenced by the nature of interface.

**Keywords**: - Monoclinic, variable, interfaces, perfect, stress-free, rigid.

## INTRODUCTION

The static deformation model is useful to investigate geological fault movement and stress distribution. It is observed from the study on earthquake and earth structure (Stein and Wysession[1]) that the earth is anisotropic in nature. The problems related to Dislocation Theory have been investigated by many researchers, e.g., Steketee [2], Chinnery [3], [4], Maruyama [5],[6]. This theory has been proved very useful to study the ground deformation field produced by faulting. The sites of most earthquakes are along geological faults which are surfaces of material discontinuity in the Earth. The critical region where faulting often takes place is near or at the interface boundary and this region may be visualized as a two-phase medium of infinite extent and such a model has been used in both fundamental and applied research. Although, at present, the half-space model is considered to be adequate for most applications, the two phase model is useful in considering the effect of internal structural discontinuity in the Earth by ignoring the effect of free surface of the Earth.

The static deformation of semi-infinite elastic isotropic half space and two welded isotropic half spaces due to a very long strike slip fault have been studied by many investigators (e.g.,

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Maruyama [6], Savage [7], Sharma et al [8], Rani & Singh [9]) and others. Garg et al [10] obtained the analytical solutions for the deformation of an orthotropic layered half space due to long strike slip fault. Madan et al [11],[12] studied the deformation due to different slip profiles (uniform, parabolic, linear, cubic and elliptic) along a very long vertical strike slip fault in two phase orthotropic media.

Ting [13] derived the Green's functions for a two-phase monoclinic elastic media line force and screw dislocations for anti-plane deformation for a monoclinic elastic medium. Singh et al [14] obtained closed form analytical expressions due to uniform slip along a long inclined strike slip fault located monoclinic elastic half-space. By using the results of Ting [13], Kumar et al [15] obtained the analytical solutions for the horizontal displacement due to uniform slip along a long strike slip fault situated in two phase monoclinic elastic media. A problem with the uniform slip model is that it predicts stress singularities around the edges of the fault. Furthermore, uniform slip is sometimes not sufficient to explain complicated surface deformation i.e. uniform slip models cannot be used in the near fields, e.g., vertical movements associated with strike slip faulting.

In order to study these phenomenons, it is necessary to consider the discontinuity which varies over the fault plane, and in the particular, one that decreased slowly to zero near the extremities of the zone of movement. The Nankaido Earthquake (1707) is a strike-slip fault with variable slip. To describe an earthquake source, slip and fault length are the most readly observed parameters. Effect of variable slip and stress drop can be studied more easily in two dimensions. In two dimensions, a very long fault can be modeled as a distribution of elastic dislocation lines, strike slip dislocations when slip is parallel to the long fault dimension and dip slip dislocations when perpendicular.

Recently, Madan et al [16] considered exponential slip along strike-slip fault situated in monoclinic elastic half-space. It was observed that the anisotropy and various dislocations affect the deformation significantly.

In continuation of our previous work, Madan et al [16], we have obtained the explicit expressions for the displacements at any point of two homogeneous monoclinic elastic half spaces with different interfaces due to variable slip of exponential type along a very long strike slip dislocation situated in the lower elastic half space. A particular case is also considered when the upper media is orthotropic.

## PROBLEM FORMULATIONS

Consider a homogeneous anisotropic elastic infinite medium consisting of two elastic half-spaces. The upper half space $y < 0$ is termed as medium II and the lower half space $y > 0$ is termed as medium I with $y$ −axis vertically downwards. The origin of the Cartesian coordinate system oxyz is placed on the interface. We further assume that both elastic medium are homogeneous and monoclinic with $z = 0$ as the symmetry plane. The interface may either be perfect, stress free or rigid.

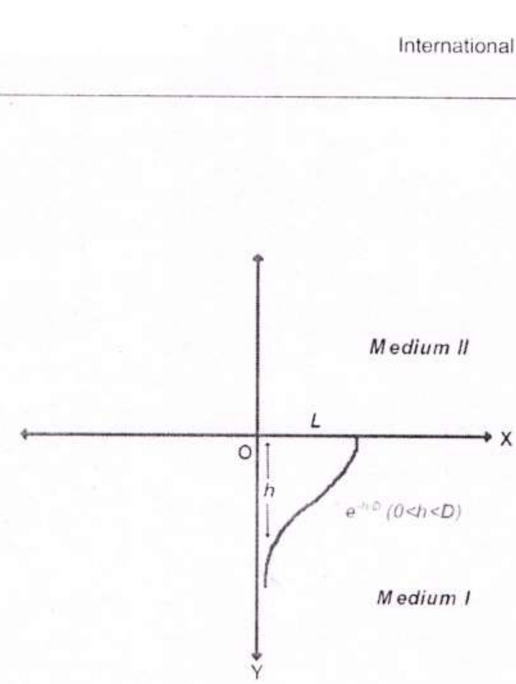


Fig. 1 Geometry of the Problem

We consider a vertical strike slip fault of infinite width L and infinite length $(-\infty < y < \infty)$ with exponential variable slip along the fault which lies at the interface in the lower half-space. The considered problem is anti-plane strain problem in which the displacement components $(u, v, w)$ in the direction of $(x, y, z)$ are independent of z coordinate so that $u = v = 0$, $w = w(x, y)$.

Following Ting [13] and Kumar *et al* [15], when the contact between two monoclinic elastic half-spaces is perfect, the displacements at any point due to a long vertical strike slip fault situated in lower monoclinic elastic medium along the fault are given by

$$w^{(1)} = \frac{1}{2\pi\gamma^{(1)}}\int_D b\{-C_{55}^{(1)}[x+\varepsilon_1(y-h)]\left(\frac{1}{R_1^2}-\frac{K}{S_1^2}\right) - C_{45}^{(1)}[\varepsilon_1\{x+\varepsilon_1(y-h)\}\left(\frac{1}{R_1^2}-\frac{K}{S_1^2}\right) + \alpha_1^2\left(\frac{y-h}{R_1^2}+K\frac{y+h}{S_1^2}\right)]\}ds \tag{1}$$

$$w^{(2)} = \frac{1+K}{2\pi\gamma^{(2)}}\int_D b\{-C_{55}^{(1)}(x+\varepsilon_2 y-\varepsilon_1 h) - C_{45}^{(1)}\ [\varepsilon_1(x+\varepsilon_2 y-\varepsilon_1 h)+\alpha_1(\alpha_2 y-\alpha_1 h)]\}\frac{1}{R_2^2}ds \tag{2}$$

where $b$ is the slip on the fault which may be uniform or variable

$$\begin{gathered}
R_1^2 = (x+\varepsilon_1(y-h))^2 + (\alpha_1(y-h))^2 \\
S_1^2 = (x+\varepsilon_1(y-h))^2 + (\alpha_1(y+h))^2 \\
R_2^2 = (x+\varepsilon_1 y-\varepsilon_1 h)^2 + (\alpha_2 y-\alpha_1 h)^2 \\
\varepsilon_1 = -C_{45}^{(1)}/C_{44}^{(1)}, \quad \varepsilon_2 = -C_{45}^{(2)}/C_{44}^{(2)} \\
\gamma_1 = C_{55}^{(1)}/C_{44}^{(1)}, \quad \gamma_2 = C_{55}^{(2)}/C_{44}^{(2)} \\
\alpha_1 = (\gamma_1-\varepsilon_1^2)^{1/2}, \quad \alpha_2 = (\gamma_2-\varepsilon_2^2)^{1/2} \\
K = \frac{m^{(2)}-m^{(1)}}{m^{(2)}+m^{(1)}},\ m^{(n)} = [C_{44}^{(n)}C_{55}^{(n)} - C_{45}^{(n)^2}]^{1/2}
\end{gathered} \tag{3}$$

## PROBLEM SOLUTIONS

Let the slip $b$ varies with depth $D$ exponentially, say

$$b(h) = b_0 \exp\left(-{}^{h}/_{D}\right) \tag{4}$$

Or

$$b(h) = b_0 \left(1 - \frac{h}{D} + \frac{h^2}{2D^2}\right) + O\left(\frac{h}{D}\right)^3, \ \ (0 < h < D). \tag{5}$$

Since, $\frac{h}{D} < 1$, therefore we neglect the higher power terms of $\frac{h}{D}$ ($\geq 3$). From (1), (2), (5) and using Wolfram Mathematica, the explicit expressions for the displacement at any point of two-phase monoclinic elastic media are obtained as:

$$w^{(1)} = -\frac{b_0}{4\pi\gamma_1^2}[(2\gamma_1^2(1-\gamma_1 Y) - 2\gamma_1\varepsilon_1 X(\gamma_1 - Y) + \gamma_1^2 Y^2 + \varepsilon_1^2 X^2 - \alpha_1^2 X^2)\tan^{-1}\frac{\alpha_1 X}{Y^2 - \gamma_1 Y(1-Y) + \varepsilon_1 X(2Y-1)} + K(-2\gamma_1^2(1+\gamma_1 Y) + 2\gamma_1\varepsilon_1(X + 2\varepsilon_1 Y)(\gamma_1 - Y) + \gamma_1^2 Y^2 + (\varepsilon_1^2 - \alpha_1^2)(X + 2\varepsilon Y)^2)\tan^{-1}\frac{\alpha_1(X+2\varepsilon_1 Y)}{(X+2\varepsilon_1 Y)^2 - \gamma_1 Y(1+Y) - \varepsilon_1(X+2\varepsilon Y)(2Y+1)} - \alpha_1 X(\gamma_1(1-Y) - \varepsilon_1 X)\log\frac{(\gamma_1 - \gamma_1 Y - \varepsilon_1 X)^2 + \alpha_1^2 X^2}{(\gamma_1 Y + \varepsilon_1 X)^2 + \alpha_1^2 X^2} + K\alpha_1(X + 2\varepsilon_1 Y)(\gamma_1(1-Y) + \varepsilon_1(X + 2\varepsilon_1 Y))\log\frac{(\gamma_1 + \gamma_1 Y - \varepsilon_1(X+2\varepsilon_1 Y))^2 + \alpha_1^2 X^2 + 4\varepsilon_1\alpha_1 XY + 4\varepsilon_1^2\alpha_1^2 Y^2}{(\gamma_1 Y - \varepsilon_1(X+2\varepsilon_1 Y))^2 + \alpha_1^2 X^2 + 4\varepsilon_1\alpha_1 XY + 4\varepsilon_1^2\alpha_1^2 Y^2} + \gamma_1\alpha_1(X + K(X + 2\varepsilon_1 Y))] \tag{6}$$

and

$$w^{(2)} = \frac{(1-K)b_0}{4\pi\gamma_1^2}[(2\gamma_1^2 - 2\gamma_1(\varepsilon_1(\varepsilon_1 Y + X) + \alpha_1\alpha_2 Y) + \varepsilon_1^2(\varepsilon_2 Y + X)^2 + 4\varepsilon_1\alpha_1\alpha_2 Y(\varepsilon_2 Y + X) + \alpha_1^2(-(Y^2(\varepsilon_2^2 - \alpha_2^2) + 2\varepsilon_2 XY + X^2)) - \varepsilon_1^2\alpha_2^2 Y^2)(\tan^{-1}\frac{\gamma_1 - \varepsilon_1(\varepsilon_2 Y + X) - \alpha_1\alpha_2 Y}{\varepsilon_1\alpha_2 Y - \alpha_1(\varepsilon_2 Y + X)} + \tan^{-1}\frac{\varepsilon_1(\varepsilon_2 Y + X) + \alpha_1\alpha_2 Y}{\varepsilon_1\alpha_2 Y - \alpha_1(\varepsilon_2 Y + X)}) + (Y(\alpha_2\varepsilon_1 - \alpha_1\varepsilon_2) - \alpha_1 X)(\varepsilon_1(\varepsilon_2 Y + X) + \alpha_1\alpha_2 Y - \gamma_1)\log(\frac{(\gamma_1 - \varepsilon_1(\varepsilon_2 Y + X) - \alpha_1\alpha_2 Y)^2 + \varepsilon_2^2\alpha_1^2 Y^2 + 2\varepsilon_2\alpha_1^2 XY - 2\varepsilon_1\varepsilon_2\alpha_1\alpha_2 Y^2 + \alpha_1^2 X^2 - 2\varepsilon_1\alpha_1\alpha_2 XY + \varepsilon_1^2\alpha_2^2 Y^2}{(\varepsilon_1(\varepsilon_2 Y + X) - \alpha_1\alpha_2 Y)^2 + \varepsilon_2^2\alpha_1^2 Y^2 + 2\varepsilon_2\alpha_1^2 XY - 2\varepsilon_1\varepsilon_2\alpha_1\alpha_2 Y^2 + \alpha_1^2 X^2 - 2\varepsilon_1\alpha_1\alpha_2 XY + \varepsilon_1^2\alpha_2^2 Y^2})] \tag{7}$$

## PARTICULAR CASES:

a) When the medium I is orthotropic, we take $\varepsilon_1 = 0$ in equation (6) and obtain the corresponding displacement:

$$w^{(1)} = -\frac{b_0}{4\pi\gamma_1}[(2\gamma_1(1-\gamma_1 Y) + \gamma_1 Y^2 - X^2)\tan^{-1}\frac{\sqrt{\gamma_1}X}{Y^2 - \gamma_1 Y(1-Y)} + K(-2\gamma_1(1+\gamma_1 Y) + \gamma_1 Y^2 - X^2)\tan^{-1}\frac{\sqrt{\gamma_1}X}{X^2 - \gamma_1 Y(1+Y)} - \sqrt{\gamma_1}X(1-Y)\log\frac{\gamma_1(1-Y)^2 + X^2}{\gamma_1 Y^2 + X^2} + K\sqrt{\gamma_1}X(1+Y)\log\frac{\gamma_1(1+Y)^2 + X^2}{\gamma_1 Y^2 + X^2} + \sqrt{\gamma_1}X(1+K)] \tag{8}$$

b) On taking K = -1 and K = 1in equation (6), the displacement of monoclinic elastic half space with a free boundary and with a rigid boundary can be obtained respectively.

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## NUMERICAL RESULTS AND DISCUSSIONS

In this section, we wish to examine the effect of exponential discontinuity and anisotropic parameters of an elastic medium due to a very long vertical strike slip fault of finite width and infinite length situated in lower anisotropic elastic half-space.

For numerical computations, we use the values of monoclinic elastic constants for Dolomite material Madan *et al* [12], orthotropic elastic constants for Olivine and Topaz material given by Verma [17] & Love [18] and transversely isotropic elastic constants for graphite & isotropic elastic constants for glass given by Madan *et al* [13] . Define the dimensionless quantities through the relations

$$W = \frac{w}{D},\ X = \frac{x}{D},\ Y = \frac{y}{D} \tag{9}$$

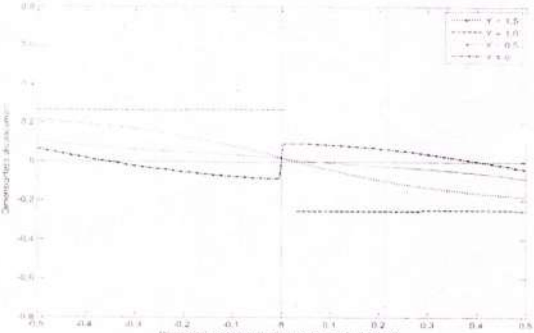

Fig. 2 Variation of the dimensionless displacement ($W^{(1)}$) with the dimensionless horizontal distance from the fault trace (X) for $\gamma = 1.043$, $\varepsilon = -0.063$, K = 0.644(Topaz-Dolomite)

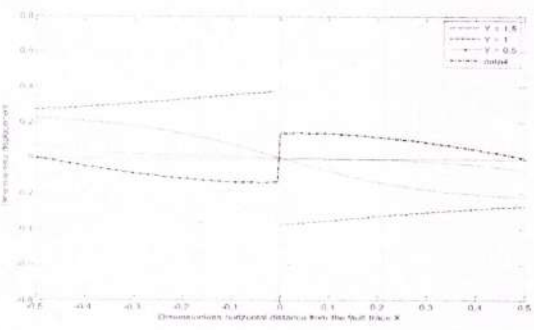

Fig. 3 Variation of the dimensionless displacement ($W^{(1)}$) with the dimensionless horizontal distance from the fault trace (X) for $\gamma = 1.3696$, $\varepsilon = 0$, K = 0.4581(Topaz-Olivine)

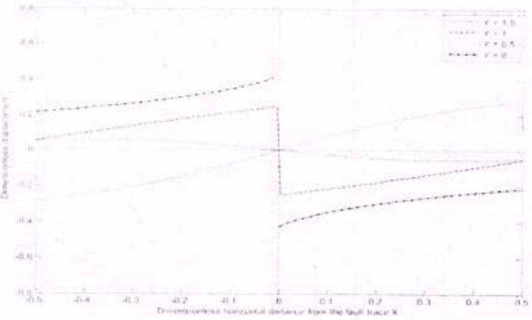

Fig. 4 Variation of the dimensionless displacement ($w^{(1)}$) with the dimensionless horizontal distance from the fault trace (X) for $\gamma = 1$, $\varepsilon = 0$, K = -0.7179(Topaz-Graphite)

and define the material models as MII-MI i.e, when Topaz is lying over Dolomite we represent as Topaz-Dolomite etc. Figures (2)-(4) show the variation of dimensionless displacements for Topaz-Dolomite model at different depth levels $Y = 0, Y = 0.5, Y = 1, Y = 1.5$ for unbounded $(K = 0)$ medium, stress free $(K = 1)$ and rigid $(K = -1)$ surface respectively with $\gamma = 1$ and $\varepsilon = 0.2$.

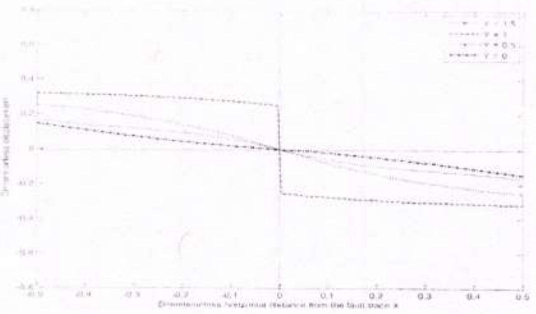

Fig. 5 Variation of the dimensionless displacement ($w^{(1)}$) with the dimensionless horizontal distance from the fault trace (X) for $\gamma = 1, \varepsilon = 0$, K = 0.99999 (Topaz-Glass)

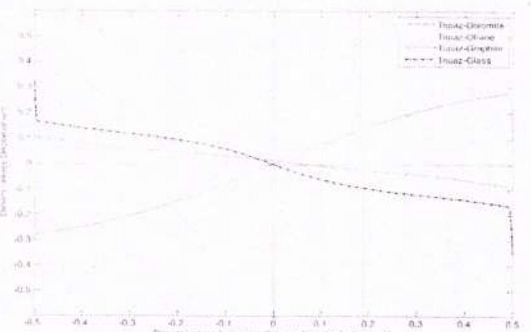

Fig. 6 Variation of the dimensionless displacement ($w^{(1)}$) with the dimensionless horizontal distance from the fault trace (X) for $Y = 0.5$

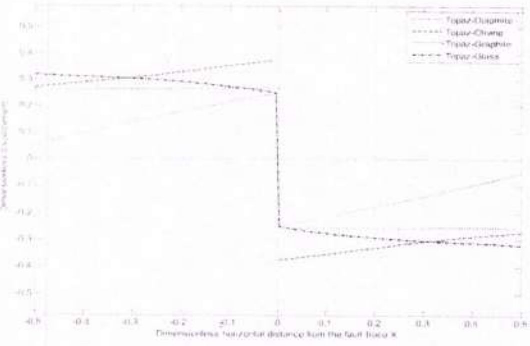

Fig. 7 Variation of the dimensionless displacement ($w^{(1)}$) with the dimensionless horizontal distance from the fault trace (X) for $Y = 1$

To examine the effect of anisotropy, we compare the dimensionless displacements for models in which upper medium II is considered Topaz(Orthotropic) and varying the lower medium I as Dolomite(monoclinic),Olivine(orthotropic), graphite(transversely isotropic) and glass(Isotropic) in figures (5)-(7) at different depth levels. From these figures it has been observed that as in the lower medium the anisotropy increases the significant effect of anisotropy measured. From these figures it is noticed that the difference in magnitude between the displacement due to Topaz-Graphite(ortho-transversely isotropic) and Topaz-Glass(ortho-iso) is greater than that of difference between the displacements due to Topaz-Dolamite(ortho-monoclinic) and Topaz-Olive(ortho-ortho). From the above figures, we observe that the displacement field is significantly influenced by the nature of interface between two elastic half spaces and anisotropy of the elastic medium.

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# SOME STRONG CONVERGENCE RESULTS OF RANDOM ITERATIVE ALGORITHMS WITH ERRORS IN BANACH SPACES

Renu Chugh, Vivek Kumar, and Satish Narwal

Abstract. In this paper, we study the strong convergence and stability of a new two step random iterative scheme with errors for accretive Lipschitzian mapping in real Banach spaces. The new iterative scheme is more acceptable because of much better convergence rate and less restrictions on parameters as compared to random Ishikawa iterative scheme with errors. We support our analytic proofs by providing numerical examples. Applications of random iterative schemes with errors to variational inequality are also given. Our results improve and establish random generalization of results obtained by Chang [4], Zhang [31] and many others.

## 1. Introduction and preliminaries

The machinery of random fixed point theory provides a convenient way of modelling many problems arising in non-linear analysis, probability theory and for solution of random equations in applied sciences. With the recent rapid developments in random fixed point theory, there has been a renewed interest in random iterative schemes [5, 6, 7, 22, 23, 24, 26]. In linear spaces, Mann and Ishikawa iterative schemes are two general iterative schemes which have been successfully applied to fixed point problems [1, 2, 13, 14]. In recent, many stability and convergence results of iterative schemes have been established, using Lipschitz accretive (or pseudo-contractive) mapping in Banach spaces [4, 8, 31]. Since in deterministic case the consideration of error terms is an important part of any iterative scheme, therefore motivated by the work of Ćirić [11, 12, 13, 14, 15], we introduce a two step random iterative scheme with errors and prove that the iterative scheme is stable with respect to $T$ with Lipschitz condition where $T$ is an accretive mapping in arbitrary real Banach space.

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Let $X$ be a real separable Banach space and let $J$ denote the normalized duality pairing from $X$ to $2^{X^*}$ given by

$$J(x) = \{f \in X^* : \langle x, f\rangle = \|x\|\,\|f\|, \|f\| = \|x\|\}, \quad x \in X,$$

where $X^*$ denote the dual space of $X$ and $\langle \cdot, \cdot\rangle$ denote the generalized duality pairing between $X$ and $X^*$.

Suppose $(\Omega, \Sigma)$ denotes a measurable space consisting of a set $\Omega$ and sigma algebra $\Sigma$ of subsets of $\Omega$ and $C$, a nonempty subset of $X$. Then random Mann iterative scheme with errors is defined as follows:

$$x_{n+1}(w) = (1-\alpha_n)x_n(w) + \alpha_n T(w, x_n(w)) + u_n(w), \quad \text{for each } w \in \Omega,\ n \geq 0, \tag{1.1}$$

where $0 \leq \alpha_n \leq 1$, $x_0 : \Omega \to C$, an arbitrary measurable mapping and $\{u_n(w)\}$ is a sequence of measurable mappings from $\Omega$ to $C$.

Also, random Ishikawa iterative scheme with errors is defined as follows:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)x_n(w) + \alpha_n T(w, y_n(w)) + u_n(w), \\ y_n(w) &= (1-\beta_n)x_n(w) + \beta_n T(w, x_n(w)) + v_n(w), \\ &\text{for each } w \in \Omega,\ n \geq 0, \end{aligned} \tag{1.2}$$

where $0 \leq \alpha_n, \beta_n \leq 1$, $x_0 : \Omega \to C$, an arbitrary measurable mapping and $\{u_n(w)\}$, $\{v_n(w)\}$ are sequences of measurable mappings from $\Omega$ to $C$.

Obviously $\{x_n(w)\}$ and $\{y_n(w)\}$ are sequences of mappings from $\Omega$ into $C$.

Also, we consider the following two step random iterative scheme with errors $\{x_n(w)\}$ defined by

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)y_n(w) + \alpha_n T(w, y_n(w)) + u_n(w), \\ y_n(w) &= (1-\beta_n)x_n(w) + \beta_n T(w, x_n(w)) + v_n(w), \\ &\text{for each } w \in \Omega,\ n \geq 0, \end{aligned} \tag{1.3}$$

where $\{u_n(w)\}$, $\{v_n(w)\}$ are sequences of measurable mappings from $\Omega$ to $C$, $0 \leq \alpha_n, \beta_n \leq 1$ and $x_0 : \Omega \to C$, an arbitrary measurable mapping.

*Remark* 1. Putting $\beta_n = 0$, $v_n = 0$ in (1.3) and (1.2), we get random Mann iterative scheme with errors.

Now we give some definitions and lemmas, which will be used in the proof of our main results.

**Definition 1.1.** A mapping $g : \Omega \to C$ is said to be measurable if $g^{-1}(B \cap C) \in \Sigma$ for every Borel subset $B$ of $X$.

**Definition 1.2.** A function $F : \Omega \times C \to C$ is said to be a random operator if $F(\cdot, x) : \Omega \to C$ is measurable for every $x \in C$.

**Definition 1.3.** A measurable mapping $p : \Omega \to C$ is said to be random fixed point of the random operator $F : \Omega \times C \to C$, if $F(w, p(w)) = p(w)$ for all $w \in \Omega$.

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**Definition 1.4.** A random operator $F : \Omega \times C \to C$ is said to be continuous if for fixed $w \in \Omega$, $F(w, \cdot) : C \to C$ is continuous.

In the sequel, $I$ denotes the identity operator on $X$, $D(T)$ and $R(T)$ denote the domain and the range of $T$, respectively.

**Definition 1.5.** Let $T : \Omega \times X \to X$ be a mapping. Then

(i) $T$ is said to be Lipschitizian, if for any $x, y \in X$ and $w \in \Omega$, there exists $L > 0$ such that

$$\|T(w, x) - T(w, y)\| \leq L\|x - y\|. \tag{1.4}$$

(ii) $T$ is said to be nonexpansive, if for any $x, y \in X$ and $w \in \Omega$,

$$\|T(w, x) - T(w, y)\| \leq \|x - y\|. \tag{1.5}$$

(iii) $T : \Omega \times X \to X$ is pseudo-contractive [8] if and only if for all $x, y \in X$, $w \in \Omega$ and for all $r > 0$ the following inequality holds:

$$\|x - y\| \leq \|(1 + r)(x - y) - r(T(w, x) - T(w, y))\| \tag{1.6}$$

or equivalently if and only if for all $x, y \in X$, there exists $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq \|x - y\|^2$$

(iv) $T$ is said to be accretive [8], if and only if for all $x, y \in X$ and for all $r > 0$ the following inequality holds:

$$\|x - y\| \leq \|x - y + r(T(w, x) - T(w, y))\| \tag{1.7}$$

or equivalently if and only if for all $x, y \in X$, there exists $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \geq 0.$$

(v) If $T$ is accretive and $R(I + \lambda T) = X$ for any $\lambda > 0$, then $T$ is called $m$-accretive [20, 32].

Accretive mappings are connected with nonexpansive mappings. It is well known that if $T$ is accretive [10], then $(I + T)^{-1}$ is a nonexpansive single-valued mapping from $R(I + \lambda T)$ to $D(T)$. The interest in accretive mappings also stems from the following facts:

(a) If $T$ is accretive, then solutions of the equation $Tx = 0$ correspond to the equilibrium points of some evolution systems [29].

(b) Many physical problems arising in applied mathematics can be modelled in terms of initial value problem of the form:

$$\frac{dx}{dt} = -Tx, x(0) = x_0, \quad \text{where } T \text{ is an accretive mapping.}$$

(c) Their connection with the well-known class of pseudo-contractive mappings ($T$ is pseudo-contractive if and only if $I - T$ is accretive).

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Suppose that $X$ is a real reflexive Banach space, $T, A : X \to X$, $g : X \to X^*$ are three mappings, and $\varphi : X^* \to R \cup \{\infty\}$ is a function with a Gateaux differential $\partial\varphi$. Then $u$ is a solution of a variational inequality if for any given $f \in X$, there exists a $u \in X$ such that

$$g(u) \in D(\partial\varphi),$$

$$(1.8) \qquad \langle Tu - Au - f, v - g(u)\rangle \geq \varphi(g(u)) - \varphi(v) \quad \text{for all } v \in X^*.$$

**Lemma 1.6** ([4]). *Suppose $X$ is a real reflexive Banach space, $\partial\varphi \circ g : X \to 2^X$ is a mapping, then the following conclusions are equivalent:*

(i) *$x^* \in X$ is a solution of variational inclusion problem (1.8);*

(ii) *$x^* \in X$ is a fixed point of the mapping $S : X \to 2^X$;*

$$S(x) = f - (Tx - Ax + \partial\varphi(g(x))) + x;$$

(iii) *$x^* \in X$ is a solution of the equation $f = Tx - Ax + \partial\varphi(g(x))$.*

**Lemma 1.7** ([20]). *Suppose $X$ is an arbitrary real Banach space, $T : D(T) \subset X \to X$ is accretive and continuous, and $D(T) = X$. Then $T$ is m-accretive.*

**Lemma 1.8** ([32]). *Suppose $X$ is an arbitrary real Banach space, $T : D(T) \subset X \to X$ is an m-accretive mapping. Then the equation $x + Tx = f$ has a unique solution in $D(T)$ for any $f \in X$.*

**Lemma 1.9.** *Let $x_n(w)$ be a sequence of real numbers satisfying the following inequality:*

$$x_{n+1} \leq \delta x_n + \sigma_n, \quad n \geq 1,$$

*where $x_n \geq 0$, $\sigma_n \geq 0$ and $\lim_{n\to\infty} \sigma_n = 0$, $0 \leq \delta < 1$. Then $x_n \to 0$ as $n \to \infty$.*

**Definition 1.10** ([1]). Let $T : \Omega \times C \to C$ be a random operator, where $C$ is a nonempty closed convex subset of a real separable Banach space $X$. Let $x_0 : \Omega \to C$ be any measurable mapping. The sequence $\{x_{n+1}(w)\}$ of measurable mappings from $\Omega$ to $C$, for $n = 0, 1, 2, \ldots$ generated by the certain random iterative scheme involving a random operator $T$ is denoted by $\{T, x_n(w)\}$ for each $w \in \Omega$. Suppose that $x_n(w) \to p(w)$ as $n \to \infty$ for each $w \in \Omega$, where $p \in RF(T)$. Let $\{p_n(w)\}$ be any arbitrary sequence of measurable mappings from $\Omega$ to $C$. Define the sequence of measurable mappings $k_n : \Omega \to R$ by $k_n(w) = d(p_n(w), \{T, p_n(w)\})$. If for each $w \in \Omega$, $k_n(w) \to 0$ as $n \to \infty$ implies $p_n(w) \to p(w)$ as $n \to \infty$ for each $w \in \Omega$, then the random iterative scheme is said to be stable with respect to the random operator $T$.

## 2. Convergence and stability results

In this section, we establish the convergence and stability results of revised two step random iterative scheme with errors (1.3) and random Ishikawa iterative scheme with errors in real Banach spaces.

**Theorem 2.1.** *Let $X$ be a real Banach space, $T : \Omega \times X \to X$ be a Lipschitzian random mapping with a Lipschitz constant $L \geq 1$, such that $(-T)$ is accretive. Let $\{x_n(w)\}$ be the random iterative scheme with errors defined by (1.3), with the following restrictions:*

(i) $0 < \alpha < \alpha_n - L^2(1+L)\alpha_n^2 - \beta_n(L-1) < 1 \ (n \geq 0)$.

(ii) $\lim_{n\to\infty} u_n(w) = 0, \ \lim_{n\to\infty} v_n(w) = 0.$

*Then*

(I) *the sequence $\{x_n(w)\}$ converges strongly to a unique fixed point $p(w)$ of $T$.*

(II) *the sequence $\{x_n(w)\}$ is stable. Moreover, $\lim_{n\to\infty} p_n(w) = p(w)$ implies $\lim_{n\to\infty} k_n(w) = 0$.*

*Proof.* (I) From (1.3), we have

$$(x_{n+1}(w) - p(w)) - \alpha_n(T(w, x_{n+1}(w)) - T(w, p(w)))$$
$$= (1-\alpha_n)(y_n(w) - p(w)) - \alpha_n(T(w, x_{n+1}(w)) - T(w, y_n(w)) + u_n(w). \tag{2.1}$$

Since $(-T)$ is accretive and Lipschitzian mapping, so using (2.1) and (1.7), we get

$$\begin{aligned}
&\|x_{n+1}(w) - p(w)\| \\
&\leq \|x_{n+1}(w) - p(w) - \alpha_n(T(w, x_{n+1}(w)) - T(w, p(w)))\| \\
&= \|(1-\alpha_n)(y_n(w) - p(w)) - \alpha_n(T(w, x_{n+1}(w)) - T(w, y_n(w))) + u_n(w)\| \\
&\leq (1-\alpha_n)\|y_n(w) - p(w)\| + \alpha_n\|T(w, y_n(w)) - T(w, x_{n+1}(w))\| + \|u_n(w)\|.
\end{aligned} \tag{2.2}$$

Now, using Lipschitz condition on $T$, (1.3) implies

$$\begin{aligned}
&\|(T(w, x_{n+1}(w)) - T(w, y_n(w)))\| \\
&\leq L\|x_{n+1}(w) - y_n(w)\| \\
&\leq L\alpha_n\|y_n(w) - T(w, y_n(w))\| + L\|u_n(w)\| \\
&\leq L\alpha_n\|y_n(w) - p(w)\| + L\alpha_n\|T(w, y_n(w)) - p(w)\| + L\|u_n(w)\| \\
&= L\alpha_n(1+L)\|y_n(w) - p(w)\| + L\|u_n(w)\|.
\end{aligned} \tag{2.3}$$

Also, from (1.3), we have the following estimate:

$$\begin{aligned}
&\|y_n(w) - p(w)\| \\
&\leq (1-\beta_n)\|x_n(w) - p(w)\| + \beta_n\|T(w, x_n(w)) - p(w)\| + \|v_n(w)\| \\
&\leq (1-\beta_n)\|x_n(w) - p(w)\| + \beta_n L\|x_n(w) - p(w)\| + \|v_n(w)\| \\
&= [1 + \beta_n(L-1)]\|x_n(w) - p(w)\| + \|v_n(w)\|.
\end{aligned} \tag{2.4}$$

Using inequalities (2.2)-(2.4), we arrive at

$$\begin{aligned}
&\|x_{n+1}(w) - p(w)\| \\
&\leq (1-\alpha_n)[1 + \beta_n(L-1)]\|x_n(w) - p(w)\| + (1-\alpha_n)\|v_n(w)\|
\end{aligned}$$

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$$+\alpha_n^2 L(1+L)[1+\beta_n(L-1)]\|x_n(w)-p(w)\| + (1+\alpha_n L)\|u_n(w)\| + \alpha_n^2 L(1+L)\|v_n(w)\|$$
$$\leq [1+\beta_n(L-1)][1-\alpha_n+\alpha_n^2 L(1+L)]\|x_n(w)-p(w)\| + (1+L)\|u_n(w)\| + [1+L(1+L)]\|v_n(w)\|$$
$$\leq [1-\{\alpha_n-\alpha_n^2 L^2(1+L)-\beta_n(L-1)\}]\|x_n(w)-p(w)\|+(1+L)\|u_n(w)\| + [1+L(1+L)]\|v_n(w)\|$$
$$\leq [1-\alpha]\|x_n(w)-p(w)\| + (1+L)\|u_n(w)\| + [1+L(1+L)]\|v_n(w)\|. \tag{2.5}$$

Now, put $[1-\alpha]=\delta$ and $[1+L(1+L)]\|v_n(w)\| + (1+L)\|u_n(w)\| = \sigma_n$.
Then (2.5) reduces to

$$\|x_{n+1}(w)-p(w)\| \leq \delta\|x_n(w)-p(w)\| + \sigma_n.$$

Therefore, using conditions (i)-(ii) and Lemma 1.9, above inequality yields $\lim_{n\to\infty}\|x_{n+1}(w)-p(w)\| = 0$, that is $\{x_n(w)\}$ defined by (1.3) converges strongly to a random fixed point $p(w)$ of $T$.

(II) Suppose that $\{p_n(w)\} \subset X$, is an arbitrary sequence,

$$k_n(w) = \|p_{n+1}(w) - (1-\alpha_n)q_n(w) - \alpha_n T(w, q_n(w)) - u_n(w)\|,$$

where

$$q_n(w) = (1-\beta_n)p_n(w) + \beta_n T(w, p_n(w)) + v_n(w) \quad \text{and} \quad \lim_{n\to\infty} k_n(w) = 0.$$

Then

$$\|p_{n+1}(w) - T(w, p(w))\|$$
$$= \|p_{n+1}(w) - (1-\alpha_n)q_n(w) - \alpha_n T(w, q_n(w)) - u_n(w)\| + \|(1-\alpha_n)q_n(w) + \alpha_n T(w, q_n(w)) + u_n(w) - T(w, p(w))\|$$
$$= k_n(w) + \|r_n - T(w, p(w))\|, \tag{2.6}$$

where

$$r_n = (1-\alpha_n)q_n(w) + \alpha_n T(w, q_n(w)) + u_n(w). \tag{2.7}$$

Then using (2.7), we have

$$(r_n(w) - p(w)) - \alpha_n(T(w, r_n) - T(w, p(w)))$$
$$= (1-\alpha_n)(q_n(w)-p(w)) - \alpha_n(T(w, r_n(w)) - T(w, q_n(w))) + u_n(w)$$

which further implies

$$\|r_n(w) - p(w)\|$$
$$\leq \|r_n(w) - p(w) - \alpha_n(T(w, r_n(w)) - T(w, p(w)))\|$$
$$= \|(1-\alpha_n)(q_n(w)-p(w)) - \alpha_n(T(w, r_n(w)) - T(w, q_n(w))) + u_n(w)\|$$
$$\leq (1-\alpha_n)\|(q_n(w)-p(w))\| + \alpha_n\|(T(w, r_n(w)) - T(w, q_n(w)))\| + \|u_n(w)\|. \tag{2.8}$$

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Now, similar to (2.3) and (2.4), we have the following estimates:

(2.9)
$$\|(T(w, r_n(w)) - T(w, q_n(w)))\| \le L\alpha_n(1 + L)\|q_n(w) - p(w)\| + L\|u_n(w)\|,$$

(2.10)
$$\|q_n(w) - p(w)\| \le [1 + \beta_n(L - 1)]\|(p_n(w) - p(w))\| + \|v_n(w)\|.$$

Using estimates (2.8)-(2.10), we arrive at

$$\begin{aligned}&\|r_n(w) - p(w)\| \\ &\le [1 - \{\alpha_n - \alpha_n^2 L^2(1 + L) - \beta_n(L - 1)\}]\|(p_n(w) - p(w))\| \\ &\quad + (1 + L)\|u_n(w)\| + [1 + L(1 + L)]\|v_n(w)\|.\end{aligned} \tag{2.11}$$

Substituting (2.11) in (2.6), we obtain

$$\begin{aligned}&\|p_{n+1}(w) - T(w, p(w))\| \\ &\le k_n(w) + [1 - \{\alpha_n - \alpha_n^2 L^2(1 + L) - \beta_n(L - 1)\}]\|(p_n(w) - p(w))\| \\ &\quad + (1 + L)\|u_n(w)\| + [1 + L(1 + L)]\|v_n(w)\|.\end{aligned} \tag{2.12}$$

Hence again using Lemma 1.9, together with conditions (i)-(ii), (2.12) yields $\lim_{n\to\infty} p_n(w) = p(w)$.

Therefore, the iteration (1.3) is $T$-stable.

Further, let $\lim_{n\to\infty} p_n(w) = p(w)$, then using (2.11), we have

$$\begin{aligned}&k_n(w) \\ &= \|p_{n+1}(w) - (1 - \alpha_n)q_n(w) - \alpha_n T(w, q_n(w)) - u_n(w)\| \\ &= \|p_{n+1}(w) - r_n(w)\| \\ &\le \|p_n(w) - p(w)\| + \|r_n(w) - p(w)\| \\ &\le \|p_n(w) - p(w)\| + [1 - \{\alpha_n - \alpha_n^2 L^2(1 + L) - \beta_n(L - 1)\}]\|(p_n(w) - p(w))\| \\ &\quad + (1 + L)\|u_n(w)\| + [1 + L(1 + L)]\|v_n(w)\|,\end{aligned}$$

which implies $\lim_{n\to\infty} k_n(w) = 0$. This completes the proof of Theorem 2.1. □

Putting $\beta_n = 0$, in Theorem 2.1, we have the following obvious corollary:

**Corollary 2.2.** *Let $X$ be a real Banach space, $T : \Omega\times X \to X$ be a Lipschitzian random mapping with a Lipschitz constant $L \ge 1$, such that $(-T)$ is accretive. Let $\{x_n(w)\}$ be the random Mann iterative scheme with errors defined by (1.1) with the following conditions:*

(i) $0 < \alpha < \alpha_n - L^2(1 + L)\alpha_n^2 < 1$ $(n \ge 0)$.
(ii) $\lim_{n\to\infty} u_n(w) = 0$.

*Then*

(1) *the sequence $\{x_n(w)\}$ converges strongly to a unique fixed point $p(w)$ of $T$.*

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(II) *the sequence* $\{x_n(w)\}$ *is stable. Moreover,* $\lim_{n\to\infty} p_n(w) = p(w)$ *implies* $\lim_{n\to\infty} k_n(w) = 0$.

**Theorem 2.3.** *Let $X$ be a real Banach space, $T : \Omega \times X \to X$ be Lipschitzian random mapping with a Lipschitz constant $L \geq 1$, such that $(-T)$ is accretive. Let $\{x_n(w)\}$ be the random Ishikawa iterative scheme with errors defined by (1.2) with the following restrictions:*

(i) $0 < \alpha < \alpha_n - \alpha_n L(1+L)(\alpha_n + \beta_n) - L(L^2-1)\alpha_n\beta_n < 1$ $(n \geq 0)$.

(ii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$.

*Then*

(I) *the sequence* $\{x_n(w)\}$ *converges strongly to a unique fixed point* $p(w)$ *of* $T$.

(II) *the sequence* $\{x_n(w)\}$ *is stable. Moreover,* $\lim_{n\to\infty} p_n(w) = p(w)$ *implies* $\lim_{n\to\infty} k_n(w) = 0$.

*Proof.* Using (1.2), we have

$$
\begin{aligned}
&(x_{n+1}(w) - p(w)) - \alpha_n(T(w, x_{n+1}) - T(w, p(w)))\\
(2.13)\quad &= (1-\alpha_n)(x_n(w)-p(w)) - \alpha_n(T(w, x_{n+1}(w)) - T(w, y_n(w))) + u_n(w).
\end{aligned}
$$

Using (2.13) and (1.7), we get

$$
\begin{aligned}
&\|x_{n+1}(w) - p(w)\|\\
&\leq \|x_{n+1}(w) - p(w) - \alpha_n(T(w, x_{n+1}(w)) - T(w, p(w)))\|\\
&= \|(1-\alpha_n)(x_n(w) - p(w)) - \alpha_n(T(w, x_{n+1}(w)) - T(w, y_n(w))) + u_n(w)\|\\
(2.14)\quad &\leq (1-\alpha_n)\|x_n(w) - p(w)\| + \alpha_n\|T(w, x_{n+1}(w)) - T(w, y_n(w))\| + \|u_n(w)\|.
\end{aligned}
$$

As $T$ is a Lipschitz mapping with constant $L$, so we have the following estimates:

$$
\begin{aligned}
&\|T(w, x_{n+1}(w)) - T(w, y_n(w))\|\\
&\leq L\|x_{n+1}(w) - y_n(w)\|\\
(2.15)\quad &\leq L[(1-\alpha_n)\|x_n(w) - y_n(w)\| + \alpha_n\|T(w, y_n(w)) - y_n(w)\| + \|u_n(w)\|],
\end{aligned}
$$

$$
\begin{aligned}
&\|T(w, (y_n(w))) - y_n(w)\|\\
&\leq (1+L)\|y_n(w) - p(w)\|\\
&\leq (1+L)[(1-\beta_n)\|x_n(w) - p(w)\| + \beta_n\|T(w, (x_n(w))) - x_n(w)\|\\
&\quad + \|v_n(w)\|]\\
(2.16)\quad &\leq (1+L)[1 + (L-1)\beta_n]\|x_n(w) - p(w)\| + (1+L)\|v_n(w)\|
\end{aligned}
$$

and

$$
\begin{aligned}
&\|x_n(w) - y_n(w)\| \leq \beta_n\|x_n(w) - T(w, (x_n(w)))\| + \|v_n(w)\|\\
(2.17)\quad &\leq (1+L)\beta_n\|x_n(w) - p(w)\| + \|v_n(w)\|.
\end{aligned}
$$

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Using (2.16) and (2.17), (2.15) yields

$$\|(T(w,x_{n+1}(w)) - T(w,y_n(w)))\|$$
$$\leq \{L(1+L)(1-\alpha_n)\beta_n + L(1+L)[1+(L-1)\beta_n]\alpha_n\}\,\|x_n(w) - p(w)\|$$
$$+ L(1+L\alpha_n)\|v_n(w)\| + L\|u_n(w)\|$$
$$\leq [L(1+L)(\alpha_n+\beta_n) + L(L^2-1)\alpha_n\beta_n]\|x_n(w) - p(w)\|$$
$$+ L(1+L)\|v_n(w)\| + L\|u_n(w)\|. \tag{2.18}$$

Substituting (2.18) into (2.14), we arrive at

$$\|x_{n+1}(w) - p(w)\|$$
$$\leq [1-\{\alpha_n - \alpha_n L(1+L)(\alpha_n+\beta_n) - L(L^2-1)\alpha_n\beta_n\}]\|(x_n(w)-p(w))\|$$
$$+ L(1+L)\alpha_n\|v_n(w)\| + (1+L\alpha_n)\|u_n(w)\|$$
$$\leq [1-\{\alpha_n - \alpha_n L(1+L)(\alpha_n+\beta_n) - L(L^2-1)\alpha_n\beta_n\}]\|(x_n(w)-p(w))\|$$
$$+ L(1+L)\|v_n(w)\| + (1+L)\|u_n(w)\|$$
$$\leq [1-\alpha]\|(x_n(w) - p(w))\| + L(1+L)\|v_n(w)\| + (1+L)\|u_n(w)\|. \tag{2.19}$$

Now, put $[1-\alpha] = \delta$ and $L(1+L)\|v_n(w)\| + (1+L)\|u_n(w)\| = \sigma_n$.

Then (2.19) reduces to

$$\|x_{n+1}(w) - p(w)\| \leq \delta\|x_n(w) - p(w)\| + \sigma_n.$$

Therefore, using conditions (i)-(ii) and Lemma 1.9, above inequality yields $\lim_{n\to\infty} \|x_{n+1}(w) - p(w)\| = 0$, that is $\{x_n(w)\}$ defined by (1.2) converges strongly to a random fixed point $p(w)$ of $T$.

(II) The proof of this part can hold on the same lines as in the proof of part (II) in Theorem 2.1. □

Now, we demonstrate the following example to prove the validity of our results.

**Example 2.4.** Let $\Omega = [0,2]$ and $\Sigma$ be the sigma algebra of Lebesgue's measurable subsets of $\Omega$. Take $X = R$ and define random operator $T$ from $\Omega \times X$ to $X$ as $T(w,x) = w - x$. Then the measurable mapping $\xi : \Omega \to X$ defined by $\xi(w) = \frac{w}{2}$, for every $w \in \Omega$, serve as a random fixed point of $T$. It is easy to see that the operator $T$ is a Lipschitz random operator with Lipschitz constant $L = 1$ such that $(-T)$ is accretive and $\alpha_n = \frac{1}{(1+L)^3}$, $\beta_n = \frac{1}{(1+L)^6}$, $\|u_n\| = \|v_n\| = \frac{1}{(n+1)}$ satisfies all the conditions (i)-(ii) given in Theorem 2.1. and Theorem 2.3.

*Remark* 2.5. New random iterative scheme is more acceptable as compared to random Ishikawa iterative scheme with errors due to following reasons:

(1) In deterministic case, for accretive mappings new two step iterative with errors has better convergence rate as compared to Ishikawa iterative scheme with errors.

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(2) For convergence, weak control conditions on parameters are required in new two step random iterative with errors as compared to random Ishikawa iterative scheme with errors.

*Proof.* (1) Let $T(x) = 1 - x$, $L = 1$, $\alpha_n = \frac{1}{(1+L)^n}$, $\beta_n = \frac{1}{(1+L)^n}$, $\|u_n\| = \|v_n\| = \frac{1}{(n+1)}$. Then taking initial approximation $x_0 = 1$, convergence of new two step and Ishikawa iterative schemes with errors to the fixed point 0.5 of operator $T$ is shown in the following table. From table, it is obvious that new two step iterative scheme with errors has much better convergence rate as compared to Ishikawa iterative scheme with errors.

| Number of iterations | New two step iterative scheme with errors | | Ishikawa iterative scheme with errors | |
|---|---|---|---|---|
| $N$ | $Tx_n$ | $x_{n+1}$ | $Tx_n$ | $x_{n+1}$ |
| 0 | 0 | 2.83203 | 0 | 1.9707 |
| 1 | −1.83203 | 2.19437 | −0.970703 | 1.93049 |
| 2 | −1.19437 | 1.73106 | −0.930489 | 1.89137 |
| 3 | −0.731063 | 1.39441 | −0.891374 | 1.85333 |
| 4 | −0.394444 | 1.14987 | −0.853328 | 1.81632 |
| 5 | −0.14987 | 0.972171 | −0.816323 | 1.78033 |
| 6 | 0.0278291 | 0.813062 | −0.78033 | 1.74532 |
| 7 | 0.156938 | 0.749256 | −0.745321 | 1.71127 |
| 8 | 0.250744 | 0.6811 | −0.711269 | 1.67815 |
| 9 | 0.3189 | 0.63158 | −0.678149 | 1.64593 |
| 10 | 0.36842 | 0.595601 | 0 | 1.9707 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 40 | 0.499991 | 0.500007 | 0.00118339 | 0.985177 |
| 41 | 0.499993 | 0.500005 | 0.0148229 | 0.971911 |
| 42 | 0.499995 | 0.500003 | 0.0280895 | 0.959007 |
| 43 | 0.499997 | 0.500003 | 0.0409933 | 0.946456 |
| 44 | 0.499997 | 0.500002 | 0.0535442 | 0.934248 |
| 45 | 0.499998 | 0.500001 | 0.065752 | 0.922374 |
| 46 | 0.499999 | 0.500001 | 0.077626 | 0.910825 |
| 47 | 0.499999 | 0.500001 | 0.0891753 | 0.899591 |
| 48 | 0.499999 | 0.500001 | 0.100409 | 0.888665 |
| 49 | 0.499999 | 0.5 | 0.111335 | 0.878037 |
| 50 | 0.5 | 0.5 | 0.121963 | 0.8677 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 528 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 529 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 530 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 531 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 532 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 533 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 534 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 535 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 536 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 537 | 0.5 | 0.5 | 0.499999 | 0.500001 |
| 538 | 0.5 | 0.5 | 0.499999 | 0.5 |
| 539 | 0.5 | 0.5 | 0.5 | 0.5 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

(2) If we take $L = 1$, $\alpha_n = \frac{1}{4L(1+L)+L}$, $\beta_n = \frac{1}{4L(1+L)}$, then both conditions $0 < \alpha_n - L^2(1+L)\alpha_n^2 - \beta_n(L-1) < 1$ and $0 < \alpha_n - \alpha_n L(1+L)(\alpha_n + \beta_n) - L(L^2-1)\alpha_n\beta_n < 1$, are satisfied.

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But if we take $L = 1$, $\alpha_n = \frac{1}{(1+L)^3}$ and $\beta_n = \frac{1}{(1+L)}$, then $0 < \alpha_n - L^2(1+L)\alpha_n^2 - \beta_n(L-1) < 1$ is satisfied but $0 < \alpha_n - \alpha_n L(1+L)(\alpha_n + \beta_n) - L(L^2 - 1)\alpha_n\beta_n < 1$, is not satisfied.

So,

$$0 < \alpha_n - \alpha_n L(1+L)(\alpha_n + \beta_n) - L(L^2-1)\alpha_n\beta_n < 1,$$

is stronger condition than

$$0 < \alpha_n - L^2(1+L)\alpha_n^2 - \beta_n(L-1) < 1.$$

## 3. Applications

In this section, we apply the random iterative schemes with errors to find solution of nonlinear random variational inclusion problem.

**Theorem 3.1.** *Let $T, A : \Omega \times X \to X$, $g : \Omega \times X \to X^*$ are three random operators on real reflexive Banach space $X$ and $\varphi : X^* \to R \cup \{\infty\}$, a function with continuous Gateaux differential $\partial\varphi$, such that $T - A + \partial\varphi \circ g - I : \Omega \times X \to X$ is a Lipschitzian accretive random operator with a Lipschitz constant $L \geq 1$. Define a random operator $S : \Omega \times X \to X$ by $S(w,x) = f - (T(w,x) - A(w,x) + \partial\varphi(g(w,x))) + x(w)$, where $f \in X$ is any given point. For any given $x_0(w) \in X$, let $\{x_n(w)\}$ be the random iterative scheme with errors defined by*

$$x_{n+1}(w) = (1-\alpha_n)y_n(w) + \alpha_n S(w, y_n(w)) + u_n(w),$$
$$y_n(w) = (1-\beta_n)x_n(w) + \beta_n S(w, x_n(w)) + v_n(w),$$
$$\text{for each } w \in \Omega,\ n \geq 0. \tag{3.1}$$

*where $\{u_n(w)\}$, $\{v_n(w)\}$ are measurable sequences in $X$ and $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0,1]$ satisfying the following conditions:*

(i) $0 < \alpha < \alpha_n - L^{*2}(1+L^*)\alpha_n^2 - \beta_n(L^* - 1) < 1$, $L^* = 1 + L$.

(ii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$.

*Then the iterative scheme (3.1) converges to the unique solution $x^*(w) \in X$ of the following nonlinear variational inclusion problem*

$$g(w,u) \in D(\partial\varphi),$$
$$\langle T(w,u) - A(w,u) - f, v - g(w,u)\rangle \geq \varphi(g(w,u)) - \varphi(v),$$
$$\text{for all } v \in X^*. \tag{3.2}$$

*Proof.* We shall complete the proof in two steps. In the first step, we show that nonlinear variational inclusion problem (3.2) has a unique solution $x^* \in X$. In the second step, we show that iterative scheme (3.1) converges to the unique solution.

Step 1. As $T - A + \partial\varphi \circ g - I$ is a Lipschitzian accretive mapping, so by Lemma 1.7, $T - A + \partial\varphi \circ g - I$ is $m$-accretive. Hence by Lemma 1.8, for any $f \in X$, the equation

$$f = T(w,x) - A(w,x) + \partial\varphi(g(w,x)) - I(w,x) + x(w)$$

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has a unique solution $x^*(w) \in X$. Then, using Lemma 1.6, $x^*(w) \in X$ will be the solution of nonlinear variational inclusion problem (3.2) and it is fixed point of $S$.

Step 2. Now since $T - A + \partial\varphi \circ g - I : \Omega \times X \to X$ is a Lipschitzian accretive operator with a Lipschitz constant $L \geq 1$, so $S : \Omega \times X \to X$ is also a Lipschitzian mapping with Lipschitz constant $L^* = 1 + L$, such that $(-S)$ is an accretive operator. Now, replacing $T$ by $S$ in (1.3), $L$ by $L^*$ in condition (i) of Theorem 2.1 and following the same steps as in the proof of Theorem 2.1, it is easy to see that the random iterative scheme (3.1) converges to the unique solution $x^* \in X$ of nonlinear variational inclusion problem (3.2). □

Letting $\varphi \equiv 0$, $u_n(w) = v_n(w) = 0$, in Theorem 3.1, we can obtain the following theorem.

**Theorem 3.2.** *Let $T, A : \Omega \times X \to X$, $g : \Omega \times X \to X^*$ are three random operators on real reflexive Banach space $X$, such that $T - A - I : \Omega \times X \to X$ is a Lipschitzian accretive operator with a Lipschitz constant $L \geq 1$. Define a random operator $S : \Omega \times X \to X$ by $S(w, x) = f - (T(w, x) - A(w, x)) + x(w)$, where $f \in X$ is any given point. For any given $x_0(w) \in X$, let $\{x_n(w)\}$ be the random iterative scheme defined by*

$$x_{n+1}(w) = (1 - \alpha_n)y_n(w) + \alpha_n S(w, y_n(w)),$$
$$y_n(w) = (1 - \beta_n)x_n(w) + \beta_n S(w, x_n(w)) \quad \text{for each } w \in \Omega, n \geq 0, \tag{3.3}$$

*where $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0, 1]$ satisfying the following conditions:*
(i) $0 < \alpha < \alpha_n - L^{*2}(1 + L^*)\alpha_n^2 - \beta_n(L^* - 1) < 1$,
*Then the iterative scheme (3.3) converges to the unique solution $x^* \in X$ of nonlinear variational inequality*

$$\langle T(w, u) - A(w, u) - f, v - g(w, u)\rangle \geq 0 \quad \text{for all } v \in X^*.$$

**Theorem 3.3.** *Let $T, A : \Omega \times X \to X$, $g : \Omega \times X \to X^*$ are three random operators on real reflexive Banach space $X$ and $\varphi : X^* \to R \cup \{\infty\}$, a function, with continuous Gateaux differential $\partial\varphi$, such that $T - A + \partial\varphi \circ g - I : \Omega \times X \to X$ is a Lipschitzian accretive operator with a Lipschitz constant $L \geq 1$. Define an random operator $S : \Omega \times X \to X$ by $S(w, x) = f - (T(w, x) - A(w, x) + \partial\varphi(g(w, x))) + x(w)$, where $f \in X$ is any given point. For any given $x_0(w) \in X$, let $\{x_n(w)\}$ be the random Ishikawa iterative scheme with errors defined by*

$$x_{n+1}(w) = (1 - \alpha_n)x_n(w) + \alpha_n S(w, y_n(w)) + u_n(w),$$
$$y_n(w) = (1 - \beta_n)x_n(w) + \beta_n S(w, x_n(w)) + v_n(w),$$
$$\text{for each } w \in \Omega, n \geq 0, \tag{3.4}$$

*where $\{u_n(w)\}$, $\{v_n(w)\}$ are measurable sequences in $X$ and $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0, 1]$, satisfying the following conditions:*

(i) $0 < \alpha < \alpha_n - \alpha_n L^*(1 + L^*)(\alpha_n + \beta_n) - L^*(L^2 - 1)\alpha_n\beta_n < 1$.
(ii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$.

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*Then the iterative scheme* (3.4) *converges to the unique solution* $x^* \in X$ *of nonlinear variational inclusion problem* (3.2).

*Proof.* Using Theorem 2.3 and following the same arguments as in the proof of Theorem 3.1, it is easy to show the random Ishikawa iterative scheme with errors (3.4) converges to the unique solution $x^* \in X$ of nonlinear variational inclusion problem (3.2). □

**Theorem 3.4.** *Let* $T, A : \Omega \times X \to X$, $g : \Omega \times X \to X^*$ *are three random operators on real reflexive Banach space* $X$ *and* $\varphi : X^* \to R \cup \{\infty\}$, *a function with continuous Gateaux differential* $\partial\varphi$, *such that* $T - A + \partial\varphi \circ g - I : \Omega \times X \to X$ *is a Lipschitzian accretive operator with a Lipschitz constant* $L \geq 1$. *Define a random operator* $S : \Omega \times X \to X$ *by* $S(w, x) = f - (T(w, x) - A(w, x) + \partial\varphi(g(w, x))) + x(w)$, *where* $f \in X$ *is any given point. For any given* $x_0(w) \in X$, *let* $\{x_n(w)\}$ *be the random Mann iterative scheme with errors defined by*

$$x_{n+1}(w) = (1 - \alpha_n)x_n(w) + \alpha_n S(w, x_n(w)) + u_n(w),$$

$$\text{for each } w \in \Omega,\ n \geq 0, \tag{3.5}$$

*where* $\{u_n(w)\}$ *is a measurable sequence in* $X$ *and* $\{\alpha_n\}$ *is a sequence in* $[0, 1]$ *satisfying following conditions:*

(i) $0 < \alpha < \alpha_n - L^2(1 + L)\alpha_n^2 < 1$.
(ii) $\lim_{n \to \infty} u_n(w) = 0$.

*Then the iterative scheme* (3.5) *converges to the unique solution* $x^* \in X$ *of nonlinear variational inclusion problem* (3.2).

*Remark* 3.5. Results involving random Ishikawa and Mann iterative schemes to solve variational inclusion problem (3.2) or variational inequality can be proved as special cases of Theorems 3.1-3.4.

*Remark* 3.6. Our results are generalization, improvement and extension of some of the well known results in the following sense:

1. Theorems 3.1 and 3.3 are randomization of Theorem 3.1 of Chang [4] as well as Theorem 2.1 of Zhang [31], using new convergence technique and weak restrictive condition on parameters. In fact in Theorems 3.1 and 3.3, $\alpha_n$ and $\beta_n$ need not converge to zero as in Theorem 3.1 of Chang [4] and Theorem 2.1 of Zhang [31].
2. Theorem 3.3 holds in reflexive real Banach spaces, whereas Theorem 3.1 of Chang [4] has been proved in uniformly smooth Banach spaces.
3. In Theorems 3.1 and 3.3, unlike in Theorem 3.1 of Chang [4] and Theorem 2.1 of Zhang [31],the boundedness of range of $S$ or $Sx_n$ and $Sy_n$, is not required.
4. The Ishikawa iterative scheme has been replaced with more general random Ishikawa iterative scheme with errors and more acceptable new two step random iterative scheme with errors.

5. Stability of more acceptable new two step random iterative scheme with errors has been proved in Theorem 2.1.
6. Theorem 3.1 of Chang [4], generalizes and improves the results in [16, 17, 18, 21, 27, 28, 30], so Theorems 3.1 and 3.3 extend and establish random generalization of the work of [16, 17, 18, 21, 27, 28, 30].

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has a unique solution $x^*(w) \in X$. Then, using Lemma 1.6, $x^*(w) \in X$ will be the solution of nonlinear variational inclusion problem (3.2) and it is fixed point of $S$.

Step 2. Now since $T - A + \partial\varphi \circ g - I : \Omega \times X \to X$ is a Lipschitzian accretive operator with a Lipschitz constant $L \geq 1$, so $S : \Omega \times X \to X$ is also a Lipschitzian mapping with Lipschitz constant $L^* = 1 + L$, such that $(-S)$ is an accretive operator. Now, replacing $T$ by $S$ in (1.3), $L$ by $L^*$ in condition (i) of Theorem 2.1 and following the same steps as in the proof of Theorem 2.1, it is easy to see that the random iterative scheme (3.1) converges to the unique solution $x^* \in X$ of nonlinear variational inclusion problem (3.2). □

Letting $\varphi \equiv 0$, $u_n(w) = v_n(w) = 0$, in Theorem 3.1, we can obtain the following theorem.

**Theorem 3.2.** *Let $T, A : \Omega \times X \to X$, $g : \Omega \times X \to X^*$ are three random operators on real reflexive Banach space $X$, such that $T - A - I : \Omega \times X \to X$ is a Lipschitzian accretive operator with a Lipschitz constant $L \geq 1$. Define a random operator $S : \Omega \times X \to X$ by $S(w, x) = f - (T(w, x) - A(w, x)) + x(w)$, where $f \in X$ is any given point. For any given $x_0(w) \in X$, let $\{x_n(w)\}$ be the random iterative scheme defined by*

$$x_{n+1}(w) = (1 - \alpha_n)y_n(w) + \alpha_n S(w, y_n(w)),$$

$$(3.3) \qquad y_n(w) = (1 - \beta_n)x_n(w) + \beta_n S(w, x_n(w)) \quad \text{for each } w \in \Omega, n \geq 0,$$

*where $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0, 1]$ satisfying the following conditions:*
(i) $0 < \alpha < \alpha_n - L^{*2}(1 + L^*)\alpha_n^2 - \beta_n(L^* - 1) < 1$,
*Then the iterative scheme (3.3) converges to the unique solution $x^* \in X$ of nonlinear variational inequality*

$$\langle T(w, u) - A(w, u) - f, v - g(w, u)\rangle \geq 0 \quad \text{for all } v \in X^*.$$

**Theorem 3.3.** *Let $T, A : \Omega \times X \to X$, $g : \Omega \times X \to X^*$ are three random operators on real reflexive Banach space $X$ and $\varphi : X^* \to R \cup \{\infty\}$, a function, with continuous Gateaux differential $\partial\varphi$, such that $T - A + \partial\varphi \circ g - I : \Omega \times X \to X$ is a Lipschitzian accretive operator with a Lipschitz constant $L \geq 1$. Define an random operator $S : \Omega \times X \to X$ by $S(w, x) = f - (T(w, x) - A(w, x) + \partial\varphi(g(w, x))) + x(w)$, where $f \in X$ is any given point. For any given $x_0(w) \in X$, let $\{x_n(w)\}$ be the random Ishikawa iterative scheme with errors defined by*

$$x_{n+1}(w) = (1 - \alpha_n)x_n(w) + \alpha_n S(w, y_n(w)) + u_n(w),$$

$$y_n(w) = (1 - \beta_n)x_n(w) + \beta_n S(w, x_n(w)) + v_n(w),$$

$$(3.4) \qquad \text{for each } w \in \Omega, n \geq 0,$$

*where $\{u_n(w)\}$, $\{v_n(w)\}$ are measurable sequences in $X$ and $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0, 1]$, satisfying the following conditions:*

(i) $0 < \alpha < \alpha_n - \alpha_n L^*(1 + L^*)(\alpha_n + \beta_n) - L^*(L^2 - 1)\alpha_n\beta_n < 1$.
(ii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$.

*Then the iterative scheme* (3.4) *converges to the unique solution* $x^* \in X$ *of nonlinear variational inclusion problem* (3.2).

*Proof.* Using Theorem 2.3 and following the same arguments as in the proof of Theorem 3.1, it is easy to show the random Ishikawa iterative scheme with errors (3.4) converges to the unique solution $x^* \in X$ of nonlinear variational inclusion problem (3.2). □

**Theorem 3.4.** *Let* $T, A : \Omega \times X \to X$, $g : \Omega \times X \to X^*$ *are three random operators on real reflexive Banach space* $X$ *and* $\varphi : X^* \to R \cup \{\infty\}$, *a function with continuous Gateaux differential* $\partial\varphi$, *such that* $T - A + \partial\varphi \circ g - I : \Omega \times X \to X$ *is a Lipschitzian accretive operator with a Lipschitz constant* $L \geq 1$. *Define a random operator* $S : \Omega \times X \to X$ *by* $S(w, x) = f - (T(w, x) - A(w, x) + \partial\varphi(g(w, x))) + x(w)$, *where* $f \in X$ *is any given point. For any given* $x_0(w) \in X$, *let* $\{x_n(w)\}$ *be the random Mann iterative scheme with errors defined by*

$$x_{n+1}(w) = (1 - \alpha_n)x_n(w) + \alpha_n S(w, x_n(w)) + u_n(w), \tag{3.5}$$
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4. The Ishikawa iterative scheme has been replaced with more general random Ishikawa iterative scheme with errors and more acceptable new two step random iterative scheme with errors.

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5. Stability of more acceptable new two step random iterative scheme with errors has been proved in Theorem 2.1.
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# On convergence of random iterative schemes with errors for strongly pseudo-contractive Lipschitzian maps in real Banach spaces

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Abstract

In this work, strong convergence and stability results of a three step random iterative scheme with errors for strongly pseudo-contractive Lipschitzian maps are established in real Banach spaces. Analytic proofs are supported by providing numerical examples. Applications of random iterative schemes with errors to find solution of nonlinear random equation are also given. Our results improve and establish random generalization of results obtained by Xu and Xie [Y. Xu, F. Xie, Rostock. Math. Kolloq.. **58** (2004), 93–100], Gu and Lu [F. Gu, J. Lu, Math. Commun., **9** (2004), 149–159], Liu et al. [Z. Liu, L. Zhang, S. M. Kang, Int. J. Math. Math. Sci., **31** (2002), 611–617] and many others. 

*Keywords:* Random Iterative schemes, stability, strongly pseudo-contractive maps.
*2010 MSC:* 47H10, 47H06

## 1. Introduction and Preliminaries

The machinery of fixed point theory provides a convenient way of modelling many problems arising in non-linear analysis, probability theory and for a solution of random equations in applied sciences. see [4, 9, 11, 12, 15, 17, 18, 20, 21, 25, 27, 29, 30, 31, 33, 34, 35, 36, 38, 39, 40] and references there. With the developments in random fixed point theory, there has been a renewed interest in random iterative schemes[2, 3, 7, 8, 10]. In linear spaces, Mann and Ishikawa iterative schemes are two general iterative

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schemes which have been successfully applied to fixed point problems [1, 5, 6, 13, 14, 16, 19, 26, 28, 37]. Recently, many stability and convergence results of iterative schemes have been established, using Lipschitz accretive pseudo-contractive) and Lipschitz strongly accretive (or strongly pseudo-contractive) mappings in Banach spaces [9, 10, 12, 13, 22, 23, 24, 32, 37]. Since in deterministic case the consideration of error terms is an important part of an iterative scheme, therefore, we introduce a three step random iterative scheme with errors and prove that the iterative scheme is stable with respect to T with Lipschitz condition where T is a strongly accretive mapping in arbitrary real Banach space.

Let $X$ be a real separable Banach space and let $J$ denote the normalized duality pairing from $X$ to $2^{X^*}$ given by

$$J(x) = \{f \in X^* : \langle x, f\rangle = \|x\| \, \|f\|, \|f\| = \|x\|\}, \quad x \in X,$$

where $X^*$ denote the dual space of $X$ and $\langle \cdot, \cdot \rangle$ denote the generalized duality pairing between $X$ and $X^*$.

Suppose $(\Omega, \Sigma)$ denotes a measurable space consisting of a set $\Omega$ and sigma algebra $\Sigma$ of subsets of $\Omega$ and $C$, a nonempty subset of $X$. Let $T : \Omega \times C \to C$ be a random operator, then random Mann iterative scheme with errors is defined as follows:

$$x_{n+1}(w) = (1-\alpha_n)x_n(w) + \alpha_n T(w, x_n(w)) + u_n(w), \quad \text{for each } w \in \Omega,\ n \geq 0, \tag{1.1}$$

where $0 \leq \alpha_n \leq 1$, $x_0 : \Omega \to C$, an arbitrary measurable mapping and $\{u_n(w)\}$ is a sequence of measurable mappings from $\Omega$ to $C$.

Also, random Ishikawa iterative scheme with errors is defined as follows:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)x_n(w) + \alpha_n T(w, y_n(w)) + u_n(w), \\ y_n(w) &= (1-\beta_n)x_n(w) + \beta_n T(w, x_n(w)) + v_n(w), \quad \text{for each } w \in \Omega,\ n \geq 0, \end{aligned} \tag{1.2}$$

where $0 \leq \alpha_n, \beta_n \leq 1$, $x_0 : \Omega \to C$, an arbitrary measurable mapping and $\{u_n(w)\}$, $\{v_n(w)\}$ are sequences of measurable mappings from $\Omega$ to $C$.

Obviously $\{x_n(w)\}$ and $\{y_n(w)\}$ are sequences of mappings from $\Omega$ in to $C$.

Also, we consider the following three step random iterative scheme with errors $\langle x_n(w)\rangle$ defined by

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)y_n(w) + \alpha_n T(w, y_n(w)) + u_n(w), \\ y_n(w) &= (1-\beta_n)z_n(w) + \beta_n T(w, z_n(w)) + v_n(w), \\ z_n(w) &= (1-\gamma_n)x_n(w) + \gamma_n T(w, x_n(w)) + w_n(w), \text{for each } w \in \Omega,\ n \geq 0, \end{aligned} \tag{1.3}$$

where $\{u_n(w)\}$, $\{v_n(w)\}$, $\{w_n(w)\}$ are sequences of measurable mappings from $\Omega$ to $C$, $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$ and $x_0 : \Omega \to C$, an arbitrary measurable mapping.

Putting $\beta_n = 0$, $v_n = 0$ in (1.2) and $\beta_n = 0$, $v_n = 0$, $\gamma_n = 0$, $w_n = 0$ in (1.3), we get random Mann iterative scheme with errors (1.1).

Now we give some definitions and lemmas, which will be used in the proofs of our main results.

**Definition 1.1.** A mapping $g : \Omega \to C$ is said to be measurable if $g^{-1}(B \cap C) \in \Sigma$ for every Borel subset $B$ of $X$.

**Definition 1.2.** A function $F : \Omega \times C \to C$ is said to be a random operator if $F(\cdot, x) : \Omega \to C$ is measurable for every $x \in C$.

**Definition 1.3.** A measurable mapping $p : \Omega \to C$ is said to be random fixed point of the random operator $F : \Omega \times C \to C$, if $F(w, p(w)) = p(w)$ for all $w \in \Omega$.

**Definition 1.4.** A random operator $F : \Omega \times C \to C$ is said to be continuous if for fixed $w \in \Omega$, $F(w, \cdot) : C \to C$ is continuous.

In the sequel, $I$ denotes the identity operator on $X$, $D(T)$ and $R(T)$ denote the domain and the range of $T$, respectively.

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**Definition 1.5.** Let $T : \Omega \times X \to X$ be a mapping. Then

(i) $T$ is said to be Lipschitzian, if for any $x, y \in X$ and $w \in \Omega$, there exists $L > 0$ such that

$$\|T(w,x) - T(w,y)\| \leq L\|x - y\|; \tag{1.4}$$

(ii) $T$ is said to be nonexpansive, if for any $x, y \in X$ and $w \in \Omega$,

$$\|T(w,x) - T(w,y)\| \leq \|x - y\|; \tag{1.5}$$

(iii) $T : \Omega \times X \to X$ is strongly pseudo-contractive [9, 12] if and only if for all $x, y \in X$, $w \in \Omega$ and for all $r > 0$, $k \in (0,1)$, the following inequality holds:

$$\|x - y\| \leq \|(x - y) + r[(I - T - kI)(w,x) - (I - T - kI)(w,y)]\|, \tag{1.6}$$

or equivalently iff for all $x, y \in X$, there exists $j(x - y) \in J(x - y)$, such that

$$\langle (I - T)x - (I - T)y, j(x - y)\rangle \leq k\|x - y\|^2;$$

(iv) $T$ is said to be strongly accretive [9, 12], if and only if for all $x, y \in X$ and for all $r > 0$, $k \in (0,1)$, the following inequality holds:

$$\|x - y\| \leq \|(x - y) + r[(T - kI)(w,x) - (T - kI)(w,y)]\|, \tag{1.7}$$

or equivalently iff for all $x, y \in X$, there exists $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \geq k\|x - y\|^2;$$

(v) If $T$ is accretive and $R(I + \lambda T) = X$ for any $\lambda > 0$, then $T$ is called m-accretive [25, 31].

A mapping $T : \Omega \times X \to X$ is said to be strongly pseudo-contractive if $I - T$ is strongly accretive, hence the fixed point theory for strongly accretive mappings is connected with fixed point theory for strongly pseudo-contractive mappings. It is well known that if $T$ is Lipschitz strongly pseudo-contractive mapping [11], then $T$ has a unique fixed point.

**Lemma 1.6** ([25]). *Suppose $X$ is an arbitrary real Banach space. $T : D(T) \subset X \to X$ is accretive and continuous, and $D(T) = X$. Then $T$ is m-accretive.*

**Lemma 1.7** ([31]). *Suppose $X$ is an arbitrary real Banach space, $T : D(T) \subset X \to X$ is an m-accretive mapping. Then the equation $x + Tx = f$ has a unique solution in $D(T)$ for any $f \in X$.*

**Lemma 1.8** ([13]). *Let $\{x_n\}$ be a sequence of real numbers satisfying the following inequality:*

$$x_{n+1} \leq \delta x_n + \sigma_n, \quad n \geq 1,$$

*where $x_n \geq 0$, $\sigma_n \geq 0$ and $\lim_{n\to\infty} \sigma_n = 0$, $0 \leq \delta < 1$. Then $x_n \to 0$ as $n \to \infty$.*

**Definition 1.9** ([2]). Let $T : \Omega \times C \to C$ be a random operator, where $C$ is a nonempty closed convex subset of a real separable Banach space $X$. Let $x_0 : \Omega \to C$ be any measurable mapping. The sequence $\{x_{n+1}(w)\}$ of measurable mappings from $\Omega$ to $C$, for $n = 0, 1, 2, \ldots$ generated by the certain random iterative scheme involving a random operator $T$ is denoted by $\{T, x_n(w)\}$ for each $w \in \Omega$. Suppose that $x_n(w) \to p(w)$ as $n \to \infty$ for each $w \in \Omega$, where $p \in RF(T)$. Let $\{p_n(w)\}$ be any arbitrary sequence of measurable mappings from $\Omega$ to $C$. Define the sequence of measurable mappings $k_n : \Omega \to R$ by $k_n(w) = d(p_n(w), \{T, p_n(w)\})$. If for each $w \in \Omega$, $k_n(w) \to 0$ as $n \to \infty$ implies $p_n(w) \to p(w)$ as $n \to \infty$ for each $w \in \Omega$, then the random iterative scheme is said to be stable with respect to the random operator $T$.

## 2. Convergence and Stability Results

In this section, we establish the convergence and stability results of three step random iterative scheme with errors (1.3) using strongly pseudo-contractive mapping under some parametrical restrictions.

**Theorem 2.1.** *Let $X$ be a real Banach space, $T : \Omega \times X \to X$ be a strongly pseudo-contractive Lipschitzian random mapping with a Lipschitz constant $L \geq 1$. Let $\{x_n(w)\}$ be the random iterative scheme with errors defined by (1.3), with the following restrictions:*

(i) $\beta_n(L-1) + \gamma_n(L-1)^2 + \beta_n\gamma_n(L-1)^2 < \alpha_n\{k - (2-k)\alpha_n L(1+L)\}(1-t)$, $(n \geq 0)$ ;

(ii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$, $\lim_{n\to\infty} w_n(w) = 0$.

*Then the sequence $\{x_n(w)\}$ converges strongly to a unique random fixed point $p(w)$ of $T$.*

*Proof.* From (1.3), we have

$$\begin{aligned}(x_{n+1}(w) - p(w)) + \alpha_n[(I-T-kI)x_{n+1}(w) - (I-T-kI)p(w)]\\ =(1-\alpha_n)(y_n(w) - p(w)) + \alpha_n[(I-T-kI)x_{n+1}(w)\\ + T(w, y_n(w))] - \alpha_n(I-kI)p(w) + u_n(w).\end{aligned} \tag{2.1}$$

Since $T$ is strongly pseudo-contractive and Lipschitzian mapping, so using (2.1) and (1.6), we get

$$\begin{aligned}\|x_{n+1}(w) - p(w)\| &\leq \|x_{n+1}(w) - p(w) + \alpha_n[(I-T-kI)x_{n+1}(w) - (I-T-kI)p(w)]\|\\ &\leq (1-\alpha_n)\|y_n(w) - p(w)\| + \alpha_n\|T(w, y_n(w)) - T(w, x_{n+1}(w))\|\\ &\quad + \alpha_n I(1-k)\|x_{n+1}(w) - p(w)\| + \|u_n(w)\|\\ &= (1-\alpha_n)\|y_n(w) - p(w)\| + \alpha_n\|T(w, y_n(w))\\ &\quad - T(w, x_{n+1}(w))\| + \alpha_n(1-k)\|x_{n+1}(w) - p(w)\| + \|u_n(w)\|,\end{aligned}$$

which implies

$$\begin{aligned}[1-\alpha_n(1-k)]\|x_{n+1}(w) - p(w)\| &\leq (1-\alpha_n)\|y_n(w) - p(w)\|\\ &\quad + \alpha_n\|T(w, y_n(w)) - T(w, x_{n+1}(w))\| + \|u_n(w)\|,\end{aligned}$$

or

$$\begin{aligned}\|x_{n+1}(w) - p(w)\| \leq \frac{(1-\alpha_n)}{[1-\alpha_n(1-k)]}\|y_n(w) - p(w)\| + \frac{\alpha_n}{[1-\alpha_n(1-k)]}\|T(w, y_n(w))\\ - T(w, x_{n+1}(w))\| + \frac{1}{[1-\alpha_n(1-k)]}\|u_n(w)\|.\end{aligned} \tag{2.2}$$

Now,

$$1 - \frac{1-\alpha_n}{1-\alpha_n(1-k)} = \frac{1-(1-\alpha_n k)}{1-\alpha_n(1-k)} \geq 1 - (1-\alpha_n k),$$

implies

$$\frac{1-\alpha_n}{1-\alpha_n(1-k)} \leq 1 - \alpha_n k, \tag{2.3}$$

and

$$1 - \frac{\alpha_n}{1-\alpha_n(1-k)} = \frac{1-\alpha_n(2-k)}{1-\alpha_n(1-k)} \geq 1 - \alpha_n(2-k).$$

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implies

$$\frac{\alpha_n}{1-\alpha_n(1-k)} \le \alpha_n(2-k), \tag{2.4}$$

and

$$\frac{1}{1-\alpha_n(1-k)} \le \frac{1}{k}. \tag{2.5}$$

Using (2.3), (2.4) and (2.5), (2.2) yields

$$\begin{aligned}\|x_{n+1}(w)-p(w)\| \le (1-\alpha_n k)\|y_n(w)-p(w)\| + \alpha_n(2-k)\|T(w,y_n(w))\\ - T(w,x_{n+1}(w))\| + \frac{\|u_n(w)\|}{k}.\end{aligned} \tag{2.6}$$

Now, using Lipschitz condition on $T$ and using (1.3), we get

$$\begin{aligned}\|(T(w,x_{n+1}(w)) - T(w,y_n(w)))\| &\le L\|x_{n+1}(w)-y_n(w)\|\\ &\le L\alpha_n\|y_n(w)-T(w,y_n(w))\| + L\|u_n(w)\|\\ &\le L\alpha_n\|y_n(w)-p(w)\| + L\alpha_n\|T(w,y_n(w))-p(w)\| + L\|u_n(w)\|\\ &\le L\alpha_n(1+L)\|y_n(w)-p(w)\| + L\|u_n(w)\|.\end{aligned} \tag{2.7}$$

Also, from (1.3), we have the following estimate:

$$\begin{aligned}\|y_n(w)-p(w)\| &\le (1-\beta_n)\|z_n(w)-p(w)\| + \beta_n\|T(w,z_n(w)-p(w))\| + \|v_n(w)\|\\ &\le (1-\beta_n)\|z_n(w)-p(w)\| + \beta_n L\|(z_n(w)-p(w))\| + \|v_n(w)\|\\ &= [1+\beta_n(L-1)]\|z_n(w)-p(w)\| + \|v_n(w)\|\\ &= [1+\beta_n(L-1)]\|(1-\gamma_n)x_n(w)+\gamma_n T(w,x_n(w)) + w_n(w) - p(w)\| + \|v_n(w)\|\\ &\le [1+\beta_n(L-1)][(1-\gamma_n)\|x_n(w)-p(w)\| + \gamma_n\|T(w,x_n(w))-p(w)\|]\\ &\quad + [1+\beta_n(L-1)]\|w_n(w)\| + \|v_n(w)\|\\ &\le [1+\beta_n(L-1)][(1-\gamma_n)\|x_n(w)-p(w)\| + L\gamma_n\|x_n(w)-p(w)\|] + \|v_n(w)\|\\ &\quad + [1+\beta_n(L-1)]\|w_n(w)\|\\ &= [1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)\|x_n(w)-p(w)\|\\ &\quad + \|v_n(w)\| + [1+\beta_n(L-1)]\|w_n(w)\|.\end{aligned} \tag{2.8}$$

Using estimate (2.8), (2.7) becomes

$$\begin{aligned}\|T(w,y_n(w)) - T(w,x_{n+1}(w))\| &\le L\alpha_n(1+L)[1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)\|x_n(w)-p(w)\|\\ &\quad + L\alpha_n(1+L)\|v_n(w)\| + L\|u_n(w)\|\\ &\quad + L\alpha_n(1+L)[1+\beta_n(L-1)]\|w_n(w)\|.\end{aligned} \tag{2.9}$$

Putting values of estimates (2.8) and (2.9) in (2.6), we get

$$\begin{aligned}&\|x_{n+1}(w)-p(w)\|\\ &\quad \le (1-\alpha_n k)[1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)\|x_n(w)-p(w)\|\\ &\qquad + \alpha_n^2(2-k)L(1+L)[1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)\|x_n(w)-p(w)\|\\ &\qquad + [1-\alpha_n k + L\alpha_n^2(2-k)(1+L)]\|v_n(w)\| + [L\alpha_n(2-k)+\frac{1}{k}]\|u_n(w)\|\\ &\qquad + [1-\alpha_n k + L\alpha_n^2(1+L)(2-k)][1+\beta_n(L-1)]\|w_n(w)\|\\ &\quad = \{(1-\alpha_n k)[1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)\\ &\qquad + (2-k)L\alpha_n^2(1+L)[1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)\}\|x_n(w)-p(w)\|\\ &\qquad + [1-\alpha_n k + L\alpha_n^2(2-k)(1+L)]\|v_n(w)\| + [L\alpha_n(2-k)+\frac{1}{k}]\|u_n(w)\|\\ &\qquad + [1-\alpha_n k + L\alpha_n^2(1+L)(2-k)][1+\beta_n(L-1)]\|w_n(w)\|\end{aligned}$$

$$
\begin{aligned}
&= [1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)[(1-\alpha_n k)\\
&\quad + L\alpha_n^2(2-k)(1+L)]\|x_n(w)-p(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(2-k)(1+L)]\|v_n(w)\| + [L\alpha_n(2-k)+\frac{1}{k}]\|u_n(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(1+L)(2-k)][1+\beta_n(L-1)]\|w_n(w)\|\\
&= [1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)\times[1-\alpha_n\{k-(2-k)\alpha_n L(1+L)\}]\|x_n(w)-p(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(2-k)(1+L)]\|v_n(w)\| + [L\alpha_n(2-k)+\frac{1}{k}]\|u_n(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(1+L)(2-k)][1+\beta_n(L-1)]\|w_n(w)\|\\
&\le 1-[\alpha_n\{k-(2-k)\alpha_n L(1+L)\}-\gamma_n(L-1)-\beta_n(L-1)-\gamma_n\beta_n(L-1)^2]\|x_n(w)-p(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(2-k)(1+L)]\|v_n(w)\| + [L\alpha_n(2-k)+\frac{1}{k}]\|u_n(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(1+L)(2-k)][1+\beta_n(L-1)]\|w_n(w)\|.
\end{aligned} \tag{2.10}
$$

Using condition (i) and (2.10), we have

$$
\begin{aligned}
\|x_{n+1}(w)-p(w)\| &\le 1-\alpha_n\{k-(2-k)\alpha_n L(1+L)\}\\
&\quad + \alpha_n\{k-(2-k)\alpha_n L(1+L)\}(1-t)\|x_n(w)-p(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(2-k)(1+L)]\|v_n(w)\| + [L\alpha_n(2-k)+\frac{1}{k}]\|u_n(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(1+L)(2-k)][1+\beta_n(L-1)]\|w_n(w)\|\\
&= [1-\alpha_n\{k-(2-k)\alpha_n L(1+L)\}t]\|x_n(w)-p(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(2-k)(1+L)]\|v_n(w)\|\\
&\quad + [L\alpha_n(2-k)+\frac{1}{k}]\|u_n(w)\|\\
&\quad + [1-\alpha_n k+L\alpha_n^2(1+L)(2-k)][1+\beta_n(L-1)]\|w_n(w)\|.
\end{aligned} \tag{2.11}
$$

If we let $\alpha_n \ge \alpha$, $\forall\, n \in N$, then (2.11) reduces to

$$
\begin{aligned}
\|x_{n+1}(w)-p(w)\| &\le [1-\alpha\{k-(2-k)\alpha L(1+L)\}t]\|x_n(w)-p(w)\|\\
&\quad + [1+L(2-k)(1+L)]\|v_n(w)\| + [L(2-k)\\
&\quad + \frac{1}{k}]\|u_n(w)\| + L[1+2L(1+L)]\|w_n(w)\|.
\end{aligned} \tag{2.12}
$$

Now, if we put $[1-\alpha\{k-(2-k)\alpha L(1+L)\}t] = \delta$ and

$$
[1+L(2-k)(1+L)]\|v_n(w)\| + \left[L(2-k)+\frac{1}{k}\right]\|u_n(w)\| + L[1+2L(1+L)]\|w_n(w)\| = \sigma_n,
$$

then (2.12) becomes

$$
\|x_{n+1}(w)-p(w)\| \le \delta\|x_n(w)-p(w)\| + \sigma_n\ . \tag{2.13}
$$

Therefore, using conditions (ii) and Lemma 1.8, inequality (2.13) yields $\lim_{n\to\infty}\|x_{n+1}(w)-p(w)\| = 0$, that is $\{x_n(w)\}$ defined by (1.3) converges strongly to a random fixed point $p(w)$ of $T$. □

**Theorem 2.2.** *Let $X$ be a real Banach space, $T : \Omega\times X \to X$ be a strongly pseudo-contractive Lipschitzian random mapping with a Lipschitz constant $L \ge 1$. Let $\{x_n(w)\}$ be the random iterative scheme with errors defined by (1.3), with the following restrictions:*

(i) $\beta_n(L-1)+\gamma_n(L-1)^2+\beta_n\gamma_n(L-1)^2 < \alpha_n\{k-(2-k)\alpha_n L(1+L)\}(1-t)$, $(n \ge 0)$;

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(ii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$, $\lim_{n\to\infty} w_n(w) = 0$.

*Then the sequence $\{x_n(w)\}$ is stable. Moreover, $\lim_{n\to\infty} p_n(w) = p(w)$ implies $\lim_{n\to\infty} k_n(w) = 0$.*

*Proof.* Suppose that $\{p_n(w)\} \subset X$, be an arbitrary sequence,

$$k_n(w) = \|p_{n+1}(w) - (1-\alpha_n)q_n(w) - \alpha_n T(w, q_n(w)) - u_n(w)\|,$$

where

$$q_n(w) = (1-\beta_n)r_n(w) + \beta_n T(w, r_n(w)) + v_n(w),$$
$$r_n(w) = (1-\gamma_n)p_n(w) + \gamma_n T(w, p_n(w)) + w_n(w),$$

such that $\lim_{n\to\infty} k_n(w) = 0$. Then

$$\begin{aligned}\|p_{n+1}(w) - T(w, p(w))\| &= \|p_{n+1}(w) - (1-\alpha_n)q_n(w) - \alpha_n T(w, q_n(w)) - u_n(w)\| \\ &\quad + \|(1-\alpha_n)q_n(w) + \alpha_n T(w, q_n(w)) + u_n(w) - T(w, p(w))\| \\ &= k_n(w) + \|s_n(w) - T(w, p(w))\|,\end{aligned} \tag{2.14}$$

where

$$s_n(w) = (1-\alpha_n)q_n(w) + \alpha_n T(w, q_n(w)) + u_n(w) . \tag{2.15}$$

From (2.15), we have

$$\begin{aligned}&s_n(w) - p(w) + \alpha_n[(I - T - kI)T(w, s_n(w)) - (I - T - kI)p(w)] \\ &= (1-\alpha_n)(q_n(w) - p(w)) + \alpha_n[(I - T - kI)s_n(w) + T(w, q_n(w))] - \alpha_n(I - kI)p(w) + u_n(w),\end{aligned}$$

which further implies

$$\begin{aligned}\|s_n(w) - p(w)\| &\le \|s_n(w) - p(w) + \alpha_n[(I - T - kI)s_n(w) - (I - T - kI)p(w)]\| \\ &\le (1-\alpha_n)\|(q_n(w) - p(w))\| + \alpha_n\|(T(w, q_n(w)) - T(w, s_n(w)))\| \\ &\quad + \alpha_n(1-k)\|(s_n(w) - p(w))\| + \|u_n(w)\|.\end{aligned} \tag{2.16}$$

Rearranging terms in (2.16) and using estimates (2.3)–(2.5), we get

$$\begin{aligned}\|s_n(w) - p(w)\| &\le (1-\alpha_n k)\|(q_n(w) - p(w))\| \\ &\quad + \alpha_n(2-k)\|T(w, q_n(w)) - T(w, s_n(w))\| + \frac{\|u_n(w)\|}{k}.\end{aligned} \tag{2.17}$$

Following the same procedure as in Theorem 2.1, similar to estimate (2.12), we have the following estimate

$$\begin{aligned}\|s_n(w) - p(w)\| &\le [1 - \alpha\{k - (2-k)\alpha L(1+L)\}t]\|p_n(w) - p(w)\| + [1 + L(2-k)(1+L)]\|v_n(w)\| \\ &\quad + \left[L(2-k) + \frac{1}{k}\right]\|u_n(w)\| + L[1 + 2L(1+L)]\|w_n(w)\|.\end{aligned} \tag{2.18}$$

Inequality (2.18) together with inequality (2.14) yields

$$\begin{aligned}\|p_{n+1}(w) - T(w, p(w))\| &\le [1 - \alpha\{k - (2-k)\alpha L(1+L)\}t]\|p_n(w) - p(w)\| \\ &\quad + [1 + L(2-k)(1+L)]\|v_n(w)\| + \left[L(2-k) + \frac{1}{k}\right]\|u_n(w)\| \\ &\quad + L[1 + 2L^2(1+L)]\|w_n(w)\| + k_n.\end{aligned} \tag{2.19}$$

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Putting $[1 - \alpha\{k - (2-k)\alpha L(1+L)\}t] = \delta$ and

$$[1 + L(2-k)(1+L)]\|v_n(w)\| + \left[L(2-k) + \frac{1}{k}\right]\|u_n(w)\| + L[1 + 2L(1+L)]\|w_n(w)\| + k_n = \sigma_n.$$

and using condition (ii), and Lemma 1.8, inequality (2.19) yields $\lim_{n\to\infty} \|p_{n+1}(w) - p(w)\| = 0$.
i.e $\lim_{n\to\infty} p_{n+1}(w) = p(w)$. Hence given iterative scheme is $T$ stable.

Now, let $\lim_{n\to\infty} p_n(w) = p(w)$, then using (2.18), we have

$$\begin{aligned} k_n(w) &= \|p_{n+1}(w) - (1-\alpha_n)q_n(w) - \alpha_n T(w, q_n(w)) - u_n(w)\| \\ &= \|p_{n+1}(w) - s_n(w)\| \\ &\le \|p_{n+1}(w) - p(w)\| + \|s_n(w) - p(w)\| \\ &\le \|p_{n+1}(w) - p(w)\| + [1 - \alpha\{k - (2-k)\alpha L(1+L)\}t]\|p_n(w) - p(w)\| \\ &\quad + [1 + L(2-k)(1+L)]\|v_n(w)\| + [L(2-k) + \frac{1}{k}]\|u_n(w)\| + L[1 + 2L(1+L)]\|w_n(w)\|, \end{aligned} \tag{2.20}$$

which implies $\lim_{n\to\infty} k_n(w) = 0$. □

Putting $\beta_n = 0$, $\gamma_n = 0$, in Theorem 2.1 and Theorem 2.2, we have the following obvious corollary:

**Corollary 2.3.** *Let $X$ be a real Banach space, $T : \Omega \times X \to X$ be a strongly pseudo-contractive Lipschitzian random mapping with a Lipschitz constant $L \ge 1$. Let $\{x_n(w)\}$ be the random Mann iterative scheme with errors defined by (1.1) with the following conditions:*

(i) $0 < \alpha < \alpha_n$, $(n \ge 0)$;

(ii) $\lim_{n\to\infty} u_n(w) = 0$.

*Then*

(i) *the sequence $\{x_n(w)\}$ converges strongly to unique fixed point $p(w)$ of $T$;*

(ii) *the sequence $\{x_n(w)\}$ is stable. Moreover, $\lim_{n\to\infty} p_n(w) = p(w)$ implies $\lim_{n\to\infty} k_n(w) = 0$, where $\{x_n(w)\} \subseteq X$ is an arbitrary sequence.*

Now, we demonstrate the following example to prove the validity of our results.

**Example 2.4.** Let $\Omega = \left[\frac{1}{2}, 2\right]$ and $\Sigma$ be the sigma algebra of Lebesgue's measurable subsets of $\Omega$. Take $X = R$ and define random operator $T$ from $\Omega \times X$ to $X$ as $T(w, x) = \frac{w}{x}$. Then the measurable mapping $\xi : \Omega \to X$ defined by $\xi(w) = \sqrt{w}$, for every $w \in \Omega$, serve as a random fixed point of $T$. It is easy to see that the operator $T$ is a Lipschitz random operator with Lipschitz constant $L = 4$ and strongly pseudo-contractive random operator for any $k \in (0, 1)$ and $\alpha_n = 0.0082$, $k = 0.9$, $t = 0.4$, $\beta_n = \frac{1}{(1+L)^6}$, $\gamma_n = \frac{1}{(1+L)^7}$, $\|u_n\| = \frac{1}{(n+1)^2}$, $\|v_n\| = \frac{1}{(n+2)^2}$, $\|w_n\| = \frac{1}{(n+3)^2}$ satisfies all the conditions (i) (ii) given in Theorem 2.1 and Theorem 2.2.

## 3. Convergence speed comparison

Let $\Omega = [0, 1]$ and $\Sigma$ be the sigma algebra of Lebesgue's measurable subsets of $\Omega$. Take $X = R$ and define random operator $T$ from $\Omega \times X$ to $X$ as $T(w, x) = 1 - 2\sin x$. Then the measurable mapping $\xi : \Omega \to X$ defined by $\xi(w) = 0.3376$, for every $w \in \Omega$, serve as a random fixed point of $T$. It is easy to see that the operator $T$ is a Lipschitz random operator with Lipschitz constant $L = 2$ such that $T$ is strongly pseudo-contractive and $\alpha_n = 0.002$, $\beta_n = \frac{1}{(1+L)^7}$, $\gamma_n = \frac{1}{(1+L)^8}$, $\|u_n\| = \frac{1}{(n+1)^2}$, $\|v_n\| = \frac{1}{(n+2)^2}$, $\|w_n\| = \frac{1}{(n+3)^2}$, $k = 0.9$, $r = 0.2$, $t = 0.5$ satisfies the conditions (i)-(ii) given in Theorem 2.1 and Theorem 2.2.

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New random iterative scheme with errors is more acceptable for strongly pseudo-contractive mappings because it has better convergence rate as compared to Mann and Ishikawa iterative schemes with errors:

Taking initial approximation $x_0 = 1.8$, convergence of new three step iterative scheme with errors, Ishikawa and Mann iterative schemes with errors to the fixed point **0.3376** of operator $T$ is shown in the following table. From table, it is obvious that in deterministic case new three step iterative scheme with errors has much better convergence rate as compared to Ishikawa and Mann iterative schemes with errors.

| Number of iterations | Three step iterative scheme with errors | Ishikawa iterative scheme with errors | Mann iterative scheme with errors |
|---|---|---|---|
| $n$ | $x_{n+1}$ | $x_{n+1}$ | $x_{n+1}$ |
| 1 | 1.79283 | 1.79874 | 1.7945 |
| 2 | 1.78567 | 1.79749 | 1.78902 |
| 3 | 1.77853 | 1.79623 | 1.78353 |
| 4 | 1.77139 | 1.79497 | 1.77806 |
| 5 | 1.76426 | 1.79372 | 1.77258 |
| 6 | 1.75715 | 1.79246 | 1.76712 |
| 7 | 1.75005 | 1.7912 | 1.76166 |
| 8 | 1.74296 | 1.78995 | 1.75621 |
| 9 | 1.73588 | 1.78869 | 1.75077 |
| 10 | 1.72881 | 1.78744 | 1.74533 |
| - | - | - | - |
| 1547 | 0.337601 | 0.593217 | 0.337846 |
| 1548 | 0.337601 | 0.592893 | 0.337844 |
| 1549 | 0.337601 | 0.592569 | 0.337843 |
| 1550 | **0.3376** | 0.592246 | 0.337841 |
| 1551 | **0.3376** | 0.591923 | 0.33784 |
| - | - | - | - |
| 2019 | **0.3376** | 0.47716 | 0.337601 |
| 2020 | **0.3376** | 0.47698 | 0.337601 |
| 2021 | **0.3376** | 0.4768 | 0.337601 |
| 2022 | **0.3376** | 0.47662 | **0.3376** |
| 2023 | **0.3376** | 0.47644 | **0.3376** |
| - | - | - | - |
| 8888 | **0.3376** | 0.337601 | **0.3376** |
| 8889 | **0.3376** | 0.337601 | **0.3376** |
| 8890 | **0.3376** | 0.337601 | **0.3376** |
| 8891 | **0.3376** | **0.3376** | **0.3376** |
| 8892 | **0.3376** | **0.3376** | **0.3376** |

## 4. Applications

In this section, we apply the random iterative schemes with errors to find solution of nonlinear random equation with Lipschitz strongly accretive mappings.

**Theorem 4.1.** *Suppose that $A : \Omega \times X \to X$ be a Lipschitz strongly accretive mapping. Let $x^*(w)$ be a solution of random equation $A(w,x) = f$; where $f \in X$ is any given point and $S(w,x) = f + x(w) - A(w,x)$, $\forall\, x \in X$. Consider the new three step random iterative scheme with errors defined by*

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)y_n(w) + \alpha_n S(w, y_n(w)) + u_n(w),\\ y_n(w) &= (1-\beta_n)z_n(w) + \beta_n S(w, z_n(w)) + v_n(w),\\ z_n(w) &= (1-\gamma_n)x_n(w) + \gamma_n S(w, x_n(w)) + w_n(w), \quad \text{for each } w \in \Omega,\ n \geq 0, \end{aligned} \tag{4.1}$$

*where $\{u_n(w)\}$, $\{v_n(w)\}$, $\{w_n(w)\}$ are sequences of measurable mappings from $\Omega$ to $X$, $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$ and $x_0 : \Omega \to X$, an arbitrary measurable mapping, satisfying*

(i) $\beta_n(L-1) + \gamma_n(L-1)^2 + \beta_n\gamma_n(L-1)^2 < \alpha_n\{k - (2-k)\alpha_n L(1+L)\}(1-t)$, $(n \geq 0)$

(ii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$, $\lim_{n\to\infty} w_n(w) = 0$,

*where $L \geq 1$ is Lipschitz constant of $S(w,x)$. Then*

(1) *$\{x_n(w)\}$ converges strongly to unique solution $x^*(w)$ of $A(w,x) = f$;*

(2) *It is S-stable to approximate the solution of $A(w,x) = f$; by new three step random iterative scheme with errors (4.1).*

*Proof.* Since $A(w,x)$ is Lipschitz strongly accretive mapping, so $S(w,x) = f + x(w) - A(w,x)$ is Lipschitz strongly pseudo-contractive mapping. Convergence of iterative scheme (4.1) to the fixed point $x^*(w)$ of mapping $S(w,x)$ is obvious from Theorem 2.1 and it is easy to see that $x^*(w)$ is unique fixed point of $S$ iff $x^*(w)$ is solution of random equation $A(w,x) = f$. Stability of iterative scheme (4.1) follows on the same lines as stability of iterative scheme (1.3) in Theorem 2.2. □

From Theorem 4.1, with ease we can prove the following theorem:

**Theorem 4.2.** *Suppose that $A : \Omega \times X \to X$ be a Lipschitz strongly accretive mapping. Let $x^*(w)$ be a solution of random equation $A(w,x) = f$; where $f \in X$ is any given point and $S(w,x) = f + x(w) - A(w,x)$, $\forall\, x \in X$. Consider the random Mann iterative scheme with errors defined by*

$$x_{n+1}(w) = (1-\alpha_n)y_n(w) + \alpha_n S(w, y_n(w)) + u_n(w), \text{ for each } w \in \Omega,\ n \geq 0, \tag{4.2}$$

*where $\{u_n(w)\}$ is a sequence of measurable mappings from $\Omega$ to $X$, $0 \leq \alpha_n \leq 1$ and $x_0 : \Omega \to X$, an arbitrary measurable mapping, satisfying*

(i) $\alpha < \alpha_n$ $(n \geq 0)$;

(ii) $\lim_{n\to\infty} u_n(w) = 0$,

*where $L \geq 1$ is Lipschitz constant of $S(w,x)$. Then*

(1) *$\{x_n(w)\}$ converges strongly to unique solution $x^*(w)$ of $A(w,x) = f$;*

(2) *It is S-stable to approximate the solution of $A(w,x) = f$; by random iterative scheme with errors (4.2).*

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# Random Iterative Algorithms and Almost Sure Stability in Banach Spaces

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**Abstract.** Many popular iterative algorithms have been used to approximate fixed point of contractive type operators. We define the concept of generalized $\phi$-weakly contractive random operator $T$ on a separable Banach space and establish Bochner integrability of random fixed point and almost sure stability of $T$ with respect to several random Kirk type algorithms. Examples are included to support new results and show their validity. Our work generalizes, improves and provides stochastic version of several earlier results by a number of researchers.

## 1. Introduction

Random fixed points are stochastic generalization of classical or deterministic fixed points and are required for various classes of random operators arising in physical systems (see [3, 4, 14, 15, 17]). Random fixed point theory was initiated in 1950 by Prague school of probabilists. The machinery of random fixed point theory provides a convenient way of modelling many problems arising in nonlinear analysis, probability theory and for a solution of random equations in applied sciences. The study of nonlinear operators has attracted the attention of many mathematicians in various spaces (see [2, 13–15, 18, 30, 32, 33] and references therein). Several interesting random fixed point results have been established in [4, 6, 8, 13, 15, 18, 19, 27, 34]. If the exact value of a fixed point of a mapping cannot be found, we approximate it through a convenient iterative algorithm. With the developments in random fixed point theory, there has been a renewed interest in random iterative algorithms [4, 6, 8, 13, 27, 34]. In linear spaces, Mann and Ishikawa iterative algorithms have been extensively applied to fixed point problems [5, 16, 25, 29].

Initially Mann [25] iterative algorithm was employed to approximate a fixed point of a non-expansive mapping where the Picard iterative algorithm failed to converge. In 1974, Ishikawa [16] iterative algorithm has been used to obtain convergence of a Lipschitzian pseudo-contractive operator where the Mann iterative algorithm was not applicable. Later, Noor iterative algorithm [26] was introduced to solve variational inequality problems. Recently, Phuengrattana and Suantai [28] introduced SP iterative algorithm and

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proved that it has better convergence rate as compared to Mann, Ishikawa and Noor iterative algorithms. Kirk [24], Rhoades [29] and Hussain et al. [12] studied Kirk type iterative algorithms with faster convergence rate than other existing iterative algorithms. Results on S-iterative algorithm for pseudo-contractive and contractive maps, respectively, were established by Sahu and Peturasel [31] and Kumar et al. [23].

Stability and convergence results for various iterative algorithms have been established in [1, 5–7, 9, 13, 20–22, 27, 28, 34]. Bochner integrability of fixed point is an interesting concept related to iterative algorithms and is used to solve different problems in functional analysis and probability theory. It is also used to study geometry of Banach spaces and differential equations in vector spaces (see [10] and references therein). Recently, Zhang et al. [34] studied almost sure $T$-stability of Ishikawa-type and Mann-type random algorithms for certain $\phi$-weakly contractive type random operators in the setup of a separable Banach space. They also established Bochner integrability of a random fixed point for such random operators. Very recently, Okeke and Abbas [27] introduced the notion of generalized $\phi$-weakly contractive random operator and obtained almost sure $T$-stability of random Ishikawa iterative algorithm for these operators.

We prove Bochner integrability of a random fixed point by using a verity of very general iterative algorithms like random Noor, random SP, random Kirk-Noor, random Kirk-SP for generalized $\phi$-weakly contractive operators satisfying the condition (2.5). Our results are improvement and generalization of the results of Zhang et al. [34], Aweke and Abbas [27] and give random version of many important known results.

## 2. Preliminaries

Let $\Sigma$ be a sigma algebra of subsets of a set $\Omega$ and $X$ be a separable Banach space. Throughout this paper, we assume that $(\Omega, \Sigma, \mu)$ is a complete probabilistic measure space, $(\Sigma, B(X))$ is the Borel measurable space.

A mapping $\xi : \Omega \to X$ is called (a) $X$-valued random variable if $\xi$ is $(\Sigma, B(X))$-measurable, (b) strongly $\mu$-measurable if, there exists a sequence $\{x_n\}$ of $\mu$-simple functions converging to $\xi$, $\mu$-almost everywhere. In view of separability of the Banach space $X$, the sum of two $X$-valued random variables is an $X$-valued random variable.

The following definitions and results will be needed in the sequel.

**Definition 2.1.** *A mapping $g : \Omega \to C$ is said to be measurable if $g^{-1}(B \cap C) \in \Sigma$ for every Borel subset $B$ of $X$ and nonempty subset $C$ of $X$.*

**Definition 2.2.** *A function $T : \Omega \times C \to C$ is said to be a random operator if $T(\cdot, x) : \Omega \to C$ is measurable for every $x \in C$.*

**Definition 2.3.** *A measurable mapping $p : \Omega \to C$ is said to be random fixed point of the random operator $T : \Omega \times C \to C$, if $T(w, p(w)) = p(w)$ for all $w \in \Omega$.*

We denote by $RF(T)$, the set of random fixed points of $T$.

**Definition 2.4 ([17]).** *A random variable $\xi : \Omega \to C$ is Bochner integrable if for each*

$$w \in \Omega, \quad \int_{\Omega} \|\xi(w)\| d\mu(w) < \infty, \tag{2.1}$$

*where $\|\xi(w)\|$ is a non-negative real valued random variable.*

The Bochner integral is a natural generalization of the familiar Lebesgue integral for vector-valued set functions.

**Definition 2.5 ([17]).** *A random variable $\xi$ is Bochner integrable if and only if there exists a sequence of random variables $\{\xi_n\}_{n=1}^{\infty}$ converging strongly to $\xi$ almost surely such that*

$$\lim_{n\to\infty} \int_{\Omega} \|\xi_n(w) - \xi(w)\| d\mu(w) = 0. \tag{2.2}$$

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**Definition 2.6** ([34]). *Let $C$ be a nonempty subset of a separable Banach space $X$. A random operator $T : \Omega \times C \to C$ is $\phi$-weakly contractive-type operator if, there exists a continuous, non-decreasing function $\phi : R^+ \to R^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$, $\phi(0) = 0$ and for each $x, y \in C$, $w \in \Omega$, we have*

$$\int_\Omega \|T(w,x) - T(w,y)\| d\mu(w) \leq \int_\Omega \|x - y\| d\mu(w) - \phi\left(\int_\Omega \|x - y\| d\mu(w)\right). \tag{2.3}$$

**Definition 2.7** ([27]). *A random operator $T : \Omega \times C \to C$ is generalized $\phi$-weakly contractive-type if, there exists $L(w) \geq 0$ and a continuous, non-decreasing function $\phi : R^+ \to R^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$, $\phi(0) = 0$ and for each $x, y \in C$, $w \in \Omega$, we have*

$$\int_\Omega \|T(w,x) - T(w,y)\| d\mu(w) \leq e^{L(w)\|x-y\|} \left[ \int_\Omega \|x - y\| d\mu(w) - \phi\left( \int_\Omega \|x - y\| d\mu(w) \right) \right]. \tag{2.4}$$

*Keeping in mind the above definitions, we introduce the following contractive condition.*

**Definition 2.8.** *A random operator $T : \Omega \times C \to C$ is generalized $\phi$-weakly contractive type if there exists $L(w) \geq 0$ and a continuous and non-decreasing function $\phi : R^+ \to R^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$, $\phi(0) = 0$ and for each $x, y \in C$, $w \in \Omega$, we have*

$$\int_\Omega \|T(w,x) - T(w,y)\| d\mu(w) \leq e^{L(w)\|x-T(w,x)\|} \left[ \int_\Omega \|x - y\| d\mu(w) - \phi\left( \int_\Omega \|x - y\| d\mu(w) \right) \right]. \tag{2.5}$$

*Both the conditions (2.4) and (2.5) are independent of each other. If $L(w) = 0$ for each $w \in \Omega$ in (2.4) and (2.5), then both reduce to condition (2.3).*

Motivated by the fact that three-step iterative algorithm gives better approximation [11] than one-step and two-step iterative algorithms, we consider random three-step Noor and random three-step SP iterative algorithms associated with $T$.

Let $T : \Omega \times C \to C$, be a random operator, where $C$ is a nonempty convex subset of $X$. Let $x_0 : \Omega \to C$, be an arbitrary measurable mapping, $\{u_n(w)\}$, $\{v_n(w)\}$, $\{w_n(w)\}$ be sequences of measurable mappings from $\Omega \to C$ and $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$. The random versions of various iterative algorithms of $T$ are defined below:

Random Noor iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)x_n(w) + \alpha_n T(w, y_n(w)) + u_n(w) \\ y_n(w) &= (1-\beta_n)x_n(w) + \beta_n T(w, z_n(w)) + v_n(w) \\ z_n(w) &= (1-\gamma_n)x_n(w) + \gamma_n T(w, x_n(w)) + w_n(w), \end{aligned} \tag{RN}$$

Random SP iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)y_n(w) + \alpha_n T(w, y_n(w)) + u_n(w) \\ y_n(w) &= (1-\beta_n)z_n(w) + \beta_n T(w, z_n(w)) + v_n(w) \\ z_n(w) &= (1-\gamma_n)x_n(w) + \gamma_n T(w, x_n(w)) + w_n(w), \end{aligned} \tag{RSP}$$

Random Ishikawa iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)\, x_n(w) + \alpha_n T\,(w, y_n(w)) + u_n(w) \\ y_n(w) &= (1-\beta_n)\, x_n(w) + \beta_n T\,(w, x_n(w)) + v_n(w) \end{aligned} \tag{RI}$$

Random S-iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= T\,(w, y_n(w)) + u_n(w) \\ y_n(w) &= (1-\beta_n)\, x_n(w) + \beta_n T\,(w, x_n(w)) + v_n(w) \end{aligned} \tag{RS}$$

Random Mann iterative algorithm with errors $\{x_n(w)\}$:

$$x_{n+1}(w) = (1-\alpha_n)x_n(w) + \alpha_n T(w, x_n(w)) + u_n(w) \tag{RM}$$

**Remark 2.9.** *Putting $\beta_n = \gamma_n = v_n(w) = w_n(w) = 0$ and $\gamma_n = w_n(w) = 0$, for all $n \in N$, in (RN), respectively, we obtain (RM) and (RI). Also, putting $\beta_n = \gamma_n = v_n(w) = w_n(w) = 0$ for all $n \in N$, in (RSP), we obtain (RM). Similarly, putting $\alpha_n = 1$, $\gamma_n = w_n(w) = 0$ and $\alpha_n = 1$, $\gamma_n = w_n(w) = 0$ in (RN) and (RSP), we get (RS).*

Hence, (RN) and (RSP) iterative algorithms are more general than (RM) and (RS) iterative algorithms. However, (RSP) iterative algorithm is most useful among all these in view of its fastness and simplicity.

For $\alpha_{n,0} \neq 0$, $\beta_{n,0} \neq 0$, $\gamma_{n,0} \neq 0$, $\alpha_{n,i}, \beta_{n,j}, \gamma_{n,k} \in [0,1]$ and fixed integers $r, s, t$, most general random Kirk type iterative algorithms are defined below:

Random Kirk-Noor iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= \alpha_{n,0}x_n(w) + \sum_{i=1}^{r} \alpha_{n,i}T^i(w, y_n) + u_n(w), \quad \sum_{i=0}^{r} \alpha_{n,i} = 1, \\ y_n(w) &= \beta_{n,0}x_n(w) + \sum_{j=1}^{s} \beta_{n,j}T^j(w, z_n) + v_n(w), \quad \sum_{j=0}^{s} \beta_{n,j} = 1 \\ z_n(w) &= \sum_{k=0}^{t} \gamma_{n,k}T^k(w, x_n) + w_n(w), \quad \sum_{k=0}^{t} \gamma_{n,k} = 1 \end{aligned} \tag{RKN}$$

Random Kirk-SP iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= \sum_{i=0}^{r} \alpha_{n,i}T^i(w, y_n) + u_n(w), \quad \sum_{i=0}^{r} \alpha_{n,i} = 1 \\ y_n(w) &= \sum_{j=0}^{s} \beta_{n,j}T^j(w, z_n) + v_n(w), \quad \sum_{j=0}^{s} \beta_{n,j} = 1 \\ z_n(w) &= \sum_{k=0}^{t} \gamma_{n,k}T^k(w, x_n) + w_n(w), \quad \sum_{k=0}^{t} \gamma_{n,k} = 1, \end{aligned} \tag{RKSP}$$

Random Kirk-Ishikawa iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= \alpha_{n,0}x_n(w) + \sum_{i=1}^{r} \alpha_{n,i}T^i(w, y_n) + u_n(w), \quad \sum_{i=0}^{r} \alpha_{n,i} = 1 \\ y_n(w) &= \beta_{n,0}x_n(w) + \sum_{j=1}^{s} \beta_{n,j}T^j(w, x_n) + v_n(w), \quad \sum_{j=0}^{s} \beta_{n,j} = 1 \end{aligned} \tag{RKI}$$

Random Kirk-S iterative algorithm with errors $\{x_n(w)\}$:

$$\begin{aligned} x_{n+1}(w) &= \sum_{i=1}^{r} \alpha_{n,i}T^i(w, y_n) + u_n(w), \quad \sum_{i=1}^{r} \alpha_{n,i} = 1 \\ y_n(w) &= \beta_{n,0}x_n(w) + \sum_{j=1}^{s} \beta_{n,j}T^j(w, x_n) + v_n(w), \quad \sum_{j=0}^{s} \beta_{n,j} = 1 \end{aligned} \tag{RKS}$$

**Remark 2.10.** *Put $r = s = t = 1$ in (RKN) and (RKSP) iterative algorithms and get (RN) and (RSP) iterative algorithms, respectively, with $\alpha_{n,1} = \alpha_n$, $\beta_{n,1} = \beta_n$, $\gamma_{n,1} = \gamma_n$.*

Define a random iterative algorithm with the help of the functions $x_n(w)$ as follows:

$$x_{n+1}(w) = f(T; x_n(w)), \quad n = 0, 1, 2, 3, \ldots, \tag{2.6}$$

where $f$ is some function measurable in the second variable.

**Definition 2.11 ([34]).** *Let $\xi^*(w)$ be a random fixed point of a random operator T and Bochner integrable with respect to the sequence $\{x_n(w)\}$. Let $\{y_n(w)\}$ be an arbitrary sequence of random variables. Set*

$$\varepsilon_n(w) = \|y_{n+1}(w) - f(T; y_n(w))\| \tag{2.7}$$

*and assume that $\|\varepsilon_n(w)\| \in L^1(\Omega(\Sigma, \mu))$, $n = 0, 1, 2, 3, \ldots$. The iterative algorithm (2.7) is almost surely T-stable if and only if $\lim_{n\to\infty} \int_\Omega \|\varepsilon_n(w)\| d\mu(w) = 0$ implies that $\xi^*(w)$ is Bochner integrable with respect to $\{y_n(w)\}$.*

**Lemma 2.12 ([5, 27]).** *Let $\{\delta_n\}$ and $\{\lambda_n\}$ be two sequences of non-negative real numbers, $\{\sigma_n\}$ be a sequence of positive numbers satisfying the conditions:*

$$\sum_{n=1}^{\infty} \sigma_n = \infty \quad \text{and} \quad \lim_{n\to\infty} \frac{\delta_n}{\sigma_n} = 0.$$

*If $\lambda_{n+1} \le \lambda_n - \sigma_n\phi(\lambda_n) + \delta_n$ holds for each $n \ge 1$, where $\phi : R^+ \to R^+$ is a continuous and strictly increasing function with $\phi(0) = 0$, then $\{\lambda_n\}$ converges to 0 as $n \to \infty$.*

**Lemma 2.13 ([5]).** *Let $\{a_n\}$ and $\{b_n\}$ be two sequences satisfying $a_{n+1} \le a_n + b_n$ for all $n \ge 1$. If $\sum_{n=1}^{\infty} b_n < \infty$, then $\lim_{n\to\infty} a_n$ exists.*

## 3. Random Noor Type Iterative Algorithms with Errors

We begin with a technical result.

**Lemma 3.1.** *Let C be a nonempty subset of a separable Banach space X and $T : \Omega \times C \to C$ be a random operator satisfying the condition (2.5). Then, $\forall\, i \in N$ and $\forall\, x, y \in C$, we have*

$$\int_\Omega \|T^i(w, x) - T^i(w, y)\| d\mu(w) \le e^{L(w)\sum_{r=1}^{i} \|T^{r-1}(w,x) - T^r(w,x)\|} \left[ \int_\Omega \|x - y\| d\mu(w) - \phi\left( \int_\Omega \|x - y\| d\mu(w) \right) \right]. \tag{3.1}$$

*Proof.* It is based on mathematical induction on $i$.

If $i = 1$, then (3.1) becomes

$$\int_\Omega \|T(w, x) - T(w, y)\| d\mu(w) \le e^{L(w)\|x - T(w,x)\|} \left[ \int_\Omega \|x - y\| d\mu(w) - \phi\left( \int_\Omega \|x - y\| d\mu(w) \right) \right].$$

i.e., (3.1) reduces to (2.5) and the result holds.

Assume that (3.1) holds for $i = q$, $q \in N$, that is,

$$\int_\Omega \|T^q(w, x) - T^q(w, y)\| d\mu(w) \le e^{L(w)\sum_{r=1}^{q} \|T^{r-1}(w,x) - T^r(w,x)\|} \left[ \int_\Omega \|x - y\| d\mu(w) - \phi\left( \int_\Omega \|x - y\| d\mu(w) \right) \right].$$

The statement is true for $i = q + 1$ as follows:

$$\begin{aligned} \|T^{q+1}x - T^{q+1}y\| &= \|T(T^q x) - T(T^q y)\| \\ &\le e^{L(w)\|T^q(w,x) - T^{q+1}(w,x)\|} \left[ \int_\Omega \|T^q(w, x) - T^q(w, y)\| d\mu(w) - \phi\left( \int_\Omega \|T^q(w, x) - T^q(w, y)\| d\mu(w) \right) \right] \\ &\le e^{L(w)\|T^q(w,x) - T^{q+1}(w,x)\|} \int_\Omega \|T^q(w, x) - T^q(w, y)\| d\mu(w). \end{aligned}$$

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**Remark 3.2.** *If $y = p(w)$ (random fixed point of $T$), then (3.1) becomes*

$$\int_{\Omega} \|T^i(w,x) - T^i(w,p)\| d\mu(w) \le \left[ \int_{\Omega} \|x - p\| d\mu(w) - \phi\left( \int_{\Omega} \|x-p\| d\mu(w) \right) \right].$$

**Theorem 3.3.** *Let $C$ be a nonempty closed and convex subset of $X$ and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of $T$ and $\{x_n(w)\}$ be (RKN) admitting the following restrictions:*

(i) $\Sigma(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0}) = \infty$

(ii) $\alpha_{n,0} < \alpha, \beta_{n,0} < \beta, \gamma_{n,0} < \gamma$

(iii) $\lim_{n\to\infty} u_n(w) = 0, \lim_{n\to\infty} v_n(w) = 0, \lim_{n\to\infty} w_n(w) = 0.$

*Then the random fixed point $p(w)$ of $T$ is Bochner integrable.*

*Proof.* To show that $p(w)$ is Bochner integrable, we shall prove that

$$\lim_{n\to\infty} \int \|x_n(w) - p(w)\| d\mu(w) = 0$$

Using iterative algorithm (RKN) and Remark 3.2, we have

$$\begin{aligned}
&\int_{\Omega} \|x_{n+1}(w) - p(w)\| d\mu(w) \\
&\le \alpha_{n,0} \int_{\Omega} \|(x_n(w) - p(w))\| d\mu(w) + \sum_{i=1}^{r} \alpha_{n,i} \int_{\Omega} \|T^i(w, y_n) - T^i(w,p)\| + \int_{\Omega} \|u_n(w)\| d\mu(w) \\
&\le \alpha_{n,0} \int_{\Omega} \|(x_n(w) - p(w))\| d\mu(w) + \sum_{i=1}^{r} \alpha_{n,i} \left[ \int_{\Omega} \|y_n - p\| d\mu(w) - \phi\left( \int_{\Omega} \|y_n - p\| d\mu(w) \right) \right] + \int_{\Omega} \|u_n(w)\| d\mu(w) \\
&\le \alpha_{n,0} \int_{\Omega} \|(x_n(w) - p(w))\| d\mu(w) + \sum_{i=1}^{r} \alpha_{n,i} \left[ \int_{\Omega} \|y_n - p\| d\mu(w) \right] + \int_{\Omega} \|u_n(w)\| d\mu(w) \qquad (3.2)
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\int_{\Omega} \|y_n(w) - p(w)\| d\mu(w) \\
&\quad \le \beta_{n,0} \int_{\Omega} \|(x_n(w) - p(w))\| d\mu(w) + \sum_{j=1}^{s} \beta_{n,j} \left[ \int_{\Omega} \|y_n - p\| d\mu(w) \right] + \int_{\Omega} \|v_n(w)\| d\mu(w) \qquad (3.3)
\end{aligned}$$

Also, using $\sum_{k=1}^{t} \gamma_{n,k} = 1 - \gamma_{n,0}$, we have

$$\begin{aligned}
\int_{\Omega} \|z_n(w) - p(w)\| d\mu(w) &\le \gamma_{n,0} \int_{\Omega} \|x_n(w) - p(w)\| d\mu(w) \\
&\quad + \sum_{k=1}^{t} \gamma_{n,k} \left[ \int_{\Omega} \|x_n - p\| d\mu(w) - \phi\left( \int_{\Omega} \|x_n - p\| d\mu(w) \right) \right] + \int_{\Omega} \|w_n(w)\| d\mu(w) \\
&\le \int_{\Omega} \|x_n(w) - p(w)\| d\mu(w) - (1 - \gamma_{n,0}) \phi\left( \int_{\Omega} \|x_n - p\| d\mu(w) \right) + \int_{\Omega} \|w_n(w)\| d\mu(w) \qquad (3.4)
\end{aligned}$$

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Putting (3.3) and (3.4) in (3.2), we get

$$\begin{aligned}&\int_\Omega \|x_{n+1}(w) - p(w)\|\, d\mu(w)\\ &\le \int_\Omega \|(x_n(w) - p(w))\|\, d\mu(w) - (1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0})\phi\left(\int_\Omega \|x_n(w) - p(w)\|\, d\mu(w)\right)\\ &\quad + \int_\Omega \|w_n(w)\| d\mu(w) + \int_\Omega \|v_n(w)\| d\mu(w) + \int_\Omega \|u_n(w)\| d\mu(w) \end{aligned} \tag{3.5}$$

Using conditions (ii)-(iii), we obtain:

$$\lim_{n\to\infty} \frac{\int_\Omega [\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|]\, d\mu(w)}{(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0})} \le \lim_{n\to\infty} \frac{\int_\Omega [\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|]\, d\mu(w)}{(1-\alpha)(1-\beta)(1-\gamma)} = 0.$$

Now putting $\int_\Omega \|x_n(w) - p(w)\|\, d\mu(w) = \lambda_n$, $(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0}) = \sigma_n$ and $\int_\Omega [\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|]\, d\mu(w) = \delta_n$ in (3.5) and using Lemma 2.12, we get $\lim_{n\to\infty} \int_\Omega \|x_n(w) - p(w)\|\, d\mu(w) = 0$. □

**Theorem 3.4.** *Let $C$ be a nonempty closed and convex subset of $X$ and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of $T$ and $\{x_n(w)\}$ be (RKN) admitting the following restrictions:*

(i) $\Sigma(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0}) = \infty$

(ii) $\alpha_{n,0} < \alpha$, $\beta_{n,0} < \beta$, $\gamma_{n,0} < \gamma$

(iii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$, $\lim_{n\to\infty} w_n(w) = 0$.

*Then, $\{x_n\}$ is almost surely T-stable.*

*Proof.* Let $\{p_n(w)\}$ be any sequence of random variable

$$\|\varepsilon_n(w)\| = \left\| p_{n+1}(w) - \alpha_{n,0} p_n(w) + \sum_{i=1}^{r} \alpha_{n,i} T^i(w, q_n) + u_n(w) \right\|, \quad n = 0, 1, 2, \ldots, \tag{3.6}$$

where

$$\begin{aligned} q_n(w) &= \beta_{n,0} p_n(w) + \sum_{j=1}^{s} \beta_{n,j} T^j(w, r_n) + v_n(w),\\ r_n(w) &= \sum_{k=0}^{t} \gamma_{n,k} T^k(w, p_n) + w_n(w) \quad \text{and}\\ &\lim_{n\to\infty} \int_\Omega \|\varepsilon_n(w)\|\, d\mu(w) = 0. \end{aligned}$$

Now we prove that $p(w)$ is Bochner integrable with respect to the sequence $\{p_n(w)\}$.

Using (3.6) and Remark 3.2, we have

$$\begin{aligned} \int_\Omega \|p_{n+1}(w) - p(w)\| d\mu(w) &\le \int_\Omega \|p_{n+1}(w) - \alpha_{n,0} p_n(w) + \sum_{i=1}^{r} \alpha_{n,i} T^i(w, q_n) + u_n(w)\| d\mu(w)\\ &\quad + \alpha_{n,0} \int_\Omega \|(p_n(w) - p(w))\|\, d\mu(w)\\ &\quad + \sum_{i=1}^{r} \alpha_{n,i} \int_\Omega \|T^i(w, q_n) - T^i(w, p)\| + \int_\Omega \|u_n(w)\| d\mu(w) \end{aligned}$$

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$$\leq \int_\Omega \|\varepsilon_n(w)\| d\mu(w) + \alpha_{n,0} \int_\Omega \|(p_n(w) - p(w))\| \, d\mu(w)$$
$$+ \sum_{i=1}^{r} \alpha_{n,i} \left[ \int_\Omega \|q_n - p\| d\mu(w) - \phi\left( \int_\Omega \|y_n - p\| d\mu(w) \right) \right] + \int_\Omega \|u_n(w)\| d\mu(w)$$
$$\leq \int_\Omega \|\varepsilon_n(w)\| d\mu(w) + \alpha_{n,0} \int_\Omega \|(p_n(w) - p(w))\| \, d\mu(w)$$
$$+ \sum_{i=1}^{r} \alpha_{n,i} \left[ \int_\Omega \|q_n - p\| d\mu(w) \right] + \int_\Omega \|u_n(w)\| d\mu(w) \tag{3.7}$$

Also

$$\int_\Omega \|q_n(w) - p(w)\| \, d\mu(w) \leq \beta_{n,0} \int_\Omega \|(p_n(w) - p(w))\| \, d\mu(w) + \sum_{j=1}^{s} \beta_{n,j} \left[ \int_\Omega \|r_n - p\| d\mu(w) \right] + \int_\Omega \|v_n(w)\| d\mu(w)$$
$$\int_\Omega \|q_n(w) - p(w)\| \, d\mu(w) \leq \beta_{n,0} \int_\Omega \|(p_n(w) - p(w))\| \, d\mu(w) + (1 - \beta_{n,0}) \left[ \int_\Omega \|r_n - p\| d\mu(w) \right] + \int_\Omega \|v_n(w)\| d\mu(w) \tag{3.8}$$

and

$$\int_\Omega \|r_n(w) - p(w)\| \, d\mu(w) \leq \int_\Omega \|p_n(w) - p(w)\| \, d\mu(w) - (1 - \gamma_{n,0})\phi\left( \int_\Omega \|p_n - p\| d\mu(w) \right) + \int_\Omega \|w_n(w)\| d\mu(w) \tag{3.9}$$

Now estimates (3.7)-(3.9) yield:

$$\int_\Omega \|p_{n+1}(w) - p(w)\| d\mu(w)$$
$$\leq \int_\Omega \|p_n(w) - p(w)\| d\mu(w) - (1 - \alpha_{n,0})(1 - \beta_{n,0})(1 - \gamma_{n,0})\phi\left( \int_\Omega \|p_n(w) - p(w)\| \, d\mu(w) \right)$$
$$+ \int_\Omega \|w_n(w)\| d\mu(w) + \int_\Omega \|v_n(w)\| d\mu(w) + \int_\Omega \|u_n(w)\| d\mu(w) + \int_\Omega \|\varepsilon_n(w)\| d\mu(w) \tag{3.10}$$

Using $\lim_{n\to\infty} \int_\Omega \|\varepsilon_n(w)\| \, d\mu(w) = 0$ and conditions (ii)-(iii), we have

$$\lim_{n\to\infty} \frac{\int_\Omega [\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|] d\mu(w) + \int_\Omega \|\varepsilon_n(w)\| \, d\mu(w)}{(1 - \alpha_{n,0})(1 - \beta_{n,0})(1 - \gamma_{n,0})}$$
$$\leq \lim_{n\to\infty} \frac{\int_\Omega [\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|] d\mu(w) + \int_\Omega \|\varepsilon_n(w)\| \, d\mu(w)}{(1 - \alpha)(1 - \beta)(1 - \gamma)} = 0$$

Now taking $\lambda_n = \int_\Omega \|p_n(w) - p(w)\| \, d\mu(w)$, $\sigma_n = (1 - \alpha_{n,0})(1 - \beta_{n,0})(1 - \gamma_{n,0})$ and $\delta_n = \int_\Omega [\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|] d\mu(w) + \int_\Omega \|\varepsilon_n(w)\| \, d\mu(w)$ in (3.10) and using Lemma 2.12, we get $\lim_{n\to\infty} \int_\Omega \|p_n(w) - p(w)\| \, d\mu(w) = 0$.

Conversely, let $p(w)$ be Bochner integrable with respect to the sequence $\{p_n(w)\}$. Then we have

$$\int_\Omega \|\varepsilon_n(w)\| d\mu(w) = \int_\Omega \|p_{n+1}(w) - \alpha_{n,0} p_n(w) + \sum_{i=1}^{r} \alpha_{n,i} T^i(w, q_n) + u_n(w)\| d\mu(w)$$
$$\leq \int_\Omega \|(p_{n+1}(w) - p(w))\| \, d\mu(w) + \alpha_{n,0} \int_\Omega \|(p_n(w) - p(w))\| \, d\mu(w)$$

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$$+\sum_{i=1}^{r}\alpha_{n,i}\int_{\Omega}\|T^i(w,q_n)-T^i(w,p)\|+\int_{\Omega}\|u_n(w)\|d\mu(w) \tag{3.11}$$

The estimates (3.8), (3.9) and (3.11) yield:

$$\begin{aligned}&\int_{\Omega}\|\varepsilon_n(w)\|d\mu(w)\\ &\leq \int_{\Omega}\|p_{n+1}(w)-p(w)\|\,d\mu(w)-(1-\alpha_{n,0})(1-\beta_{n,0}))(1-\gamma_{n,0})(1-\gamma_{n,0})\phi\left(\int_{\Omega}\|p_n(w)-p(w)\|\,d\mu(w)\right)\\ &\quad+\int_{\Omega}\|w_n(w)\|d\mu(w)+\int_{\Omega}\|v_n(w)\|d\mu(w)+\int_{\Omega}\|u_n(w)\|d\mu(w)\end{aligned} \tag{3.12}$$

Using condition (iii) and Bochner integrability of $p(w)$, (3.12) yields $\lim_{n\to\infty}\int_{\Omega}\|\varepsilon_n(w)\|\,d\mu(w)=0$. This shows that $\{x_n(w)\}$ is almost surely $T$-stable. □

**Example 3.5.** *Let $\Omega=[0,1]$ and $\Sigma$ be the sigma algebra of Lebesgue's measurable subsets of $\Omega$. Take $X=R$, $C=[0,2]$ and define random operator $T:\Omega\times C\to C$ as $T(w,x)=\frac{w-x}{2}$. Then the measurable mapping $p:\Omega\to X$ defined by $p(w)=\frac{w}{3}$, for every $w\in\Omega$, serves as a random fixed point of $T$. Also, for $\phi(t)=\frac{t}{2}$ and $L(w)=3$, we have*

$$\begin{aligned}\int_{\Omega}\|T(w,x)-T(w,y)\|d\mu(w)&=\left|\int_{\Omega}\|x-y\|d\mu(w)-\phi\left(\int_{\Omega}\|x-y\|d\mu(w)\right)\right|\\ &\leq e^{3\|x-T(w,x)\|}\left[\int_{\Omega}\|x-y\|d\mu(w)-\phi\left(\int_{\Omega}\|x-y\|d\mu(w)\right)\right],\end{aligned}$$

*Hence $T$ satisfies the condition (2.5). Taking parameters $\alpha_{n,0}=1-\frac{n^2}{1+n^2}$, $\beta_n=1-\frac{n^3}{1+n^3}$, $\gamma_n=1-\frac{n^3}{1+n^4}$ and choosing error terms $u_n(w)=\frac{w}{(n+1)^7}$, $v_n(w)=\frac{w}{(n+1)^2}$, $w_n(w)=\frac{w}{(n+1)^3}$ we have*

$$0<\alpha_{n,0},\beta_{n,0},\gamma_{n,0}\leq 0.5,\ \ \Sigma(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0})=\Sigma\frac{n^8}{(1+n^4)(1+n^3)(1+n^2)}=\infty$$

*and $\lim_{n\to\infty}u_n(w)=0$, $\lim_{n\to\infty}v_n(w)=0$, $\lim_{n\to\infty}w_n(w)=0$. So, all the conditions of Theorem 3.3 and Theorem 3.4 are satisfied and hence the random fixed point $p(w)$ of $T(w,x)$ is Bochner integrable and (RKN) is almost surely $T$-stable.*

Special cases of Theorems 3.3 and 3.4 provide the following series of new important results for random operators.

**Theorem 3.6.** *Let $C$ be a nonempty closed and convex subset of $X$ and $T:\Omega\times C\to C$ a random operator satisfying the condition (2.5) with $RF(T)\neq\phi$. Let $p(w)$ be a random fixed point of $T$ and $\{x_n(w)\}$ be (RKI) admitting the following restrictions:*

(i) $\Sigma(1-\alpha_{n,0})(1-\beta_{n,0})=\infty$

(ii) $\alpha_{n,0}<\alpha$, $\beta_{n,0}<\beta$,

(iii) $\lim_{n\to\infty}u_n(w)=0$, $\lim_{n\to\infty}v_n(w)=0$.

*Then $p(w)$ is Bochner integrable and (RKI) is almost surely $T$-stable.*

*Proof.* Put $t=0$, $w_n(w)=0$ in the proofs of Theorems 3.3 and 3.4. □

**Theorem 3.7.** *Let $C$ be a nonempty closed and convex subset of $X$ and $T:\Omega\times C\to C$ a random operator satisfying the condition (2.5) with $RF(T)\neq\phi$. Let $p(w)$ be a random fixed point of $T$ and $\{x_n(w)\}$ be (RKS) admitting the following restrictions:*

(i) $\Sigma(1-\beta_{n,0})=\infty$

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(ii) $\beta_{n,0} < \beta$,
(iii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$.

*Then $p(w)$ is Bochner integrable and (RKS) is almost surely T-stable.*

*Proof.* Set $t = 0$, $\alpha_{n,0} = 0$, $w_n(w) = 0$ in the proofs of Theorems 3.3 and 3.4. □

**Theorem 3.8.** *Let C be a nonempty closed and convex subset of X and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of T and $\{x_n(w)\}$ be (RN) admitting the following restrictions:*

(i) $\Sigma\alpha_n\beta_n\gamma_n = \infty$
(ii) $0 < \alpha \leq \alpha_n$, $0 < \beta \leq \beta_n$ and $0 < \gamma \leq \gamma_n (n \geq 1)$
(iii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$, $\lim_{n\to\infty} w_n(w) = 0$.

*Then $p(w)$ is Bochner integrable and (RN) is almost surely T-stable.*

*Proof.* Put $r = s = t = 1$, $\alpha_{n,1} = \alpha_n$, $\alpha_{n,0} = 1 - \alpha_n$, $\beta_{n,1} = \beta_n$, $\beta_{n,0} = 1 - \beta_n$, $\gamma_{n,1} = \gamma_{n,0} = 1 - \gamma_n$ in the proofs of Theorems 3.3 and 3.4. □

**Theorem 3.9.** *Let C be a nonempty closed and convex subset of X and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of T and $\{x_n(w)\}$ be (RS) admitting the following restrictions:*

(i) $\Sigma\beta_n = \infty$
(ii) $0 < \beta \leq \beta_n (n \geq 1)$
(iii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$.

*Then $p(w)$ is Bochner integrable and (RS) is almost surely T-stable.*

*Proof.* Put $r = s = 1$, $t = 0$, $\alpha_{n,0} = 0$, $\beta_{n,1} = \beta_n$, $\beta_{n,0} = 1 - \beta_n$ in the proofs of Theorems 3.3 and 3.4. □

**Theorem 3.10 ([34]).** *Let C be a nonempty closed and convex subset of X and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of T and $\{x_n(w)\}$ be (RI) admitting the following restrictions:*

(i) $\Sigma\alpha_n\beta_n = \infty$
(ii) $0 < \alpha \leq \alpha_n$, $0 < \beta \leq \beta_n (n \geq 1)$

*Then $p(w)$ is Bochner integrable and (RI) is almost surely T-stable.*

*Proof.* Put $r = s = 1$, $t = 0$, $\alpha_{n,1} = \alpha_n$, $\alpha_{n,0} = 1 - \alpha_n$, $\beta_{n,1} = \beta_n$, $\beta_{n,0} = 1 - \beta_n$, $u_n(w) = 0$, $v_n(w) = 0$, $w_n(w) = 0$, $L = 0$ in the proofs of Theorems 3.3 and 3.4. □

We now extend Theorem 3.3 for three generalized $\phi$-weakly contractive random operators as follows:

**Theorem 3.11.** *Let C be a nonempty closed and convex subset of X and let $T_i : \Omega \times C \to C$, $i = 1, 2, 3$ be three random operators satisfying the condition (2.5) with $CRF = \bigcap_{i=1}^{3} RF(T_i) \neq \phi$. Let $p(w)$ be a common random fixed point of the random operators $\{T_i, i = 1, 2, 3\}$ and $\{x_n(w)\}$ be the random Kirk-Noor algorithm of three operators with errors defined as follows:*

$$
\begin{aligned}
x_{n+1}(w) &= \alpha_{n,0}x_n(w) + \sum_{i=1}^{r} \alpha_{n,i}T_1^i(w, y_n) + u_n(w), \quad \sum_{i=0}^{r} \alpha_{n,i} = 1,\\
y_n(w) &= \beta_{n,0}x_n(w) + \sum_{j=1}^{s} \beta_{n,j}T_2^j(w, z_n) + v_n(w), \quad \sum_{j=0}^{s} \beta_{n,j} = 1 \qquad \text{(RKNTO)}\\
z_n(w) &= \sum_{k=0}^{t} \gamma_{n,k}T_3^k(w, x_n) + w_n(w), \quad \sum_{k=0}^{t} \gamma_{n,k} = 1
\end{aligned}
$$

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*where* $\{u_n(w)\}$, $\{v_n(w)\}$, $\{w_n(w)\}$ *are sequences of measurable mappings from* $\Omega$ *to* $C$ *with* $\sum \int_\Omega u_n(w)\,d\mu(w) < \infty$, $\sum \int_\Omega v_n(w)\,d\mu(w) < \infty$, $\sum \int_\Omega w_n(w)\,d\mu(w) < \infty$ *and* $0 \le \alpha_n, \beta_n, \gamma_n \le 1$. *Then common random fixed point of the random operators* $\{T_i,\ i = 1, 2, 3\}$ *is Bochner integrable if and only if for all* $w \in \Omega$, $\liminf_{n\to\infty} \int_\Omega d(x_n(w), CRF)d\mu(w) = 0$, *where* $d(x_n(w), CRF) = \inf\{\|x_n(w) - \xi(w)\| : \xi \in CRF\}$, *provided* $\int_\Omega \|T_i(w, \xi(w)) - \xi(w)\|d\mu(w) = 0$ *implies* $\|T_i(w, \xi(w)) - \xi(w)\| = 0$.

*Proof.* The necessity is obvious and hence omitted. Now to prove the sufficiency part, we show that $\lim_{n\to\infty} \int \|x_n(w) - p(w)\|\,d\mu(w) = 0$, where $p(w) \in CRF$.

Following the same steps as in the proof of Theorem 3.3, we have the following estimate:

$$\begin{aligned}
&\int_\Omega \|x_{n+1}(w) - p(w)\|\,d\mu(w)\\
&\le \int_\Omega \|(x_n(w) - p(w))\|\,d\mu(w) - (1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0})\phi\left(\int_\Omega \|x_n(w) - p(w)\|\,d\mu(w)\right)\\
&\quad + \int_\Omega \|w_n(w)\|d\mu(w) + \int_\Omega \|v_n(w)\|d\mu(w) + \int_\Omega \|u_n(w)\|d\mu(w)\\
&\le \int_\Omega \|(x_n(w) - p(w))\|\,d\mu(w) + \int_\Omega \|w_n(w)\|d\mu(w) + \int_\Omega \|v_n(w)\|d\mu(w) + \int_\Omega \|u_n(w)\|d\mu(w)
\end{aligned} \tag{3.13}$$

It follows from (3.13), in view of $d(x_n(w), CRF) = \inf\{\|x_n(w) - \xi(w)\| : \xi \in CRF\}$:

$$\int_\Omega d(x_{n+1}(w), CRF)d\mu(w) \le \int_\Omega d(x_n(w), CRF)d\mu(w) + b_n(w), \tag{3.14}$$

where $b_n(w) = (\int_\Omega \|w_n(w)\| + \int_\Omega \|v_n(w)\| + \int_\Omega \|u_n(w)\|)d\mu(w)$.

Clearly, $\sum_{n=0}^{\infty} b_n < \infty$. So by Lemma 2.13, $\lim_{n\to\infty} \int_\Omega d(x_n(w), RF)d\mu(w)$ exists.

Therefore, using the given condition in the theorem, we have for all $w \in \Omega$,

$$\lim_{n\to\infty} \int_\Omega d(x_n(w), RF)d\mu(w) = 0.$$

Now, if $a_n = \int_\Omega \|x_n(w) - p(w)\|\,d\mu(w)$ in (3.13), then it follows that for any natural number m and for all $n \ge m$,

$$\begin{aligned}
\|a_{n+m}(w)\| &\le \|a_{n+m-1}(w)\| + b_{n+m-1}(w)\\
&\le \|a_{n+m-2}(w)\| + b_{n+m-2}(w) + b_{n+m-1}(w)\\
&\le \cdots \le \|a_n(w)\| + \sum_{k=n}^{n+m-1} b_k(w).
\end{aligned} \tag{3.15}$$

Therefore, we have

$$\begin{aligned}
\|a_{n+m}(w) - a_n(w)\| &\le \|a_n(w)\| + \sum_{k=n}^{n+m-1} b_k(w) + \|a_n(w)\|\\
&= 2\|a_n(w)\| + \sum_{k=n}^{n+m-1} b_k(w)
\end{aligned} \tag{3.16}$$

As $\sum_{n=0}^{\infty} b_n < \infty$ and $\lim_{n\to\infty} \int_\Omega d(x_n(w), CRF)d\mu(w) = 0$, so there exists $m_1 \in N$ such that for all $n \ge m_1$, we have $\int_\Omega d(x_n(w), CRF)d\mu(w) < \frac{\varepsilon}{4}$ and $\sum_{k=n}^{\infty} b_k(w) < \frac{\varepsilon}{2}$.

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Hence there exists $q \in CRF$ such that

$$\int_{\Omega} \|x_n(w) - q(w)\| d\mu(w) < \frac{\varepsilon}{4} \quad \text{for all } n \geq m_1.$$

So from (3.16), we have that for all $w \in \Omega$, for all $n \geq m_1$ and for any positive integer $m$,

$$\|a_{n+m}(w) - a_n(w)\| \leq 2\|a_n(w)\| + \sum_{k=n}^{n+m-1} b_k(w) < 2\frac{\varepsilon}{4} + \frac{\varepsilon}{2} = \varepsilon,$$

or

$$\int_{\Omega} \|x_{n+m}(w) - x_n(w)\| d\mu(w) < \varepsilon,$$

from which it follows that $\left\{\int_{\Omega} \|x_n(w)\| d\mu(w)\right\}$ is a Cauchy sequence for each $w \in \Omega$. So, $\int_{\Omega} \|x_n(w)\| d\mu(w) \to \int_{\Omega} \|\xi(w)\| d\mu(w)$ as $n \to \infty$ for each $w \in \Omega$, where $\int_{\Omega} \|\xi(w)\| d\mu(w) : \Omega \to X$, being the limit of the sequence of measurable functions is also measurable.

Now we prove that $\xi(w) \in CRF$. As for each $w \in \Omega$, $\int_{\Omega} \|x_n(w)\| d\mu(w) \to \int_{\Omega} \|\xi(w)\| d\mu(w)$, when $n \to \infty$, so there exists $m_2 \in N$ such that

$$\int_{\Omega} \|x_n(w) - \xi(w)\| d\mu(w) < \frac{\varepsilon}{4} \quad \text{for all } n \geq m_2.$$

Let $m_3 = \max\{m_1, m_2\}$. Then for all $w \in \Omega$ and $n \geq m_3$, we have

$$\begin{aligned}
&\int_{\Omega} \|T_1(w, \xi(w)) - \xi(w)\| d\mu(w) \\
&\leq \int_{\Omega} \|T_1(w, \xi(w)) - \xi^*(w)\| d\mu(w) + \int_{\Omega} \|\xi^*(w) - \xi(w)\| d\mu(w) \\
&\leq \int_{\Omega} \|\xi^*(w) - \xi(w)\| d\mu(w) - \varphi\left(\int_{\Omega} \|\xi^*(w) - \xi(w)\| d\mu(w)\right) + \int_{\Omega} \|\xi^*(w) - \xi(w)\| d\mu(w) \\
&\leq 2 \int_{\Omega} \|\xi^*(w) - \xi(w)\| d\mu(w) \\
&\leq 2 \int_{\Omega} \|\xi^*(w) - x_n(w)\| d\mu(w) + 2 \int_{\Omega} \|\xi^*(w) - x_n(w)\| d\mu(w) \\
&< 2\frac{\varepsilon}{4} + 2\frac{\varepsilon}{4} = \varepsilon
\end{aligned}$$

which yields $T_1(w, \xi(w)) = \xi(w)$ for each $w \in \Omega$. As $\xi$ is measurable, so $\xi \in RF(T_1)$. In the same way, we can show that $\xi \in RF(T_2)$ and $\xi \in RF(T_3)$. Hence we have $\xi \in CRF$. Thus common random fixed point of $T_1$, $T_2$, $T_3$ is Bochner integrable.

## 4. Random SP Type Iterative Algorithm With Errors

**Theorem 4.1.** *Let $C$ be a nonempty closed and convex subset of $X$ and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of $T$ and $\{x_n(w)\}$ be (RKSP) admitting the following restrictions:*

(i) $\Sigma(1 - \gamma_{n,0}) = \infty$ or $\Sigma(1 - \beta_{n,0}) = \infty$ or $\Sigma(1 - \alpha_{n,0}) = \infty$

(ii) $\gamma_{n,0} < \gamma$ or $\beta_{n,0} < \beta$ or $\alpha_{n,0} < \alpha$

(iii) $\lim_{n\to\infty} u_n(w) = 0$, $\lim_{n\to\infty} v_n(w) = 0$, $\lim_{n\to\infty} w_n(w) = 0$.

*Then $p(w)$ of $T$ is Bochner integrable and (RKSP) is almost surely T-stable.*

*Proof.* Using iterative algorithm (RKSP) and following the corresponding steps in the proof of Theorem 3.3, we have

$$\begin{aligned}&\int_\Omega \|x_{n+1}(w) - p(w)\|d\mu(w)\\ &\leq \alpha_{n,0}\int_\Omega \|(y_n(w) - p(w))\|d\mu(w) + \sum_{i=1}^{r}\alpha_{n,i}\left[\int_\Omega \|y_n - p\|d\mu(w)\right] + \int_\Omega \|u_n(w)\|d\mu(w)\\ &\leq \int_\Omega \|(y_n(w) - p(w))\|\,d\mu(w) + \int_\Omega \|u_n(w)\|d\mu(w) \end{aligned}\tag{4.1}$$

Similarly,

$$\int_\Omega \|y_n(w) - p(w)\|\,d\mu(w) \leq \int_\Omega \|(z_n(w) - p(w))\|d\mu(w) + \int_\Omega \|v_n(w)\|d\mu(w) \tag{4.2}$$

Also,

$$\begin{aligned}&\int_\Omega \|z_n(w) - p(w)\|\,d\mu(w)\\ &\leq \int_\Omega \|x_n(w) - p(w)\|\,d\mu(w) - (1-\gamma_{n,0})\phi\left(\int_\Omega \|x_n - p\|d\mu(w)\right) + \int_\Omega \|w_n(w)\|d\mu(w)\end{aligned}\tag{4.3}$$

Using estimates (4.1)-(4.3), we arrive at

$$\begin{aligned}&\int_\Omega \|x_{n+1}(w) - p(w)\|\,d\mu(w)\\ &\leq \int_\Omega \|(x_n(w) - p(w))\|\,d\mu(w) - (1-\gamma_{n,0})\phi\left(\int_\Omega \|x_n(w) - p(w)\|\,d\mu(w)\right)\\ &\quad + \int_\Omega \|w_n(w)\|d\mu(w) + \int_\Omega \|v_n(w)\|d\mu(w) + \int_\Omega \|u_n(w)\|d\mu(w)\end{aligned}\tag{4.4}$$

Using conditions (ii)-(iii), we have

$$\lim_{n\to\infty}\frac{\int_\Omega[\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|]d\mu(w)}{(1-\gamma_{n,0})} \leq \lim_{n\to\infty}\frac{\int_\Omega[\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|]d\mu(w)}{(1-\gamma)} = 0.$$

Now, if $\lambda_n = \int_\Omega \|x_n(w) - p(w)\|\,d\mu(w)$, $\sigma_n = 1 - \gamma_{n,0}$ and

$$\delta_n = \int_\Omega [\|w_n(w)\| + \|u_n(w)\| + \|v_n(w)\|]d\mu(w)$$

in (4.4), then using Lemma 2.12, we get $\lim_{n\to\infty}\int_\Omega \|x_n(w) - p(w)\|\,d\mu(w) = 0$.

The almost sure $T$-stability of (RKSP) can be proved as in the proof of Theorem 3.4. □

**Remark 4.2.** *As $(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0}) \leq 1-\gamma_{n,0}$ implies $\Sigma(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0}) \leq \Sigma(1-\gamma_{n,0})$, so $\Sigma(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0}) = \infty$ implies $\Sigma(1-\gamma_{n,0}) = \infty$; hence we conclude that random SP iterative algorithm with errors requires weaker restriction ($\Sigma(1-\gamma_{n,0}) = \infty$) on parameters as compared to random Noor iterative algorithm with errors which requires $\Sigma(1-\alpha_{n,0})(1-\beta_{n,0})(1-\gamma_{n,0}) = \infty$, as far as Bochner integrability of fixed point $p(w)$ is concerned.*

Special cases of Theorem 4.1 provide the following new important random fixed points results.

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**Theorem 4.3.** *Let $C$ be a nonempty closed and convex subset of $X$ and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of $T$ and $\{x_n(w)\}$ be (RSP) admitting the following restrictions:*

(i) $\Sigma \gamma_n = \infty$

(ii) $0 < \gamma \leq \gamma_n (n \geq 1)$

(iii) $\lim_{n\to\infty} u_n(w) = 0, \lim_{n\to\infty} v_n(w) = 0, \lim_{n\to\infty} w_n(w) = 0.$

*Then $p(w)$ is Bochner integrable and (RSP) is almost surely T-stable.*

*Proof.* Put $r = s = t = 1$, $\alpha_{n,1} = \alpha_n$, $\alpha_{n,0} = 1 - \alpha_n$, $\beta_{n,1} = \beta_n$, $\beta_{n,0} = 1 - \beta_n$, $\gamma_{n,1} = \gamma_{n,0} = 1 - \gamma_n$ in the proof of Theorem 4.1. □

**Theorem 4.4.** *Let $C$ be a nonempty closed and convex subset of $X$ and $T : \Omega \times C \to C$ a random operator satisfying the condition (2.5) with $RF(T) \neq \phi$. Let $p(w)$ be a random fixed point of $T$ and $\{x_n(w)\}$ be (RM) admitting the following restrictions:*

(i) $\Sigma \alpha_n = \infty$

(ii) $0 < \alpha \leq \alpha_n (n \geq 1)$

(iii) $\lim_{n\to\infty} u_n(w) = 0.$

*Then $p(w)$ is Bochner integrable and (RM) is almost surely T-stable.*

*Proof.* Put $r = 1$, $s = t = 0$, $\alpha_{n,1} = \alpha_n$, $\alpha_{n,0} = 1 - \alpha_n$ in the proof of Theorem 4.1. □

**Theorem 4.5.** *Let $C$ be a nonempty closed and convex subset of $X$ and let $T_i : \Omega \times C \to C$, $i = 1, 2, 3$ be three random operators satisfying the condition (2.5) with $CRF = \bigcap_{i=1}^{3} RF(T_i) \neq \phi$. Let $p(w)$ be a common random fixed point of the random operators $\{T_i, i = 1, 2, 3\}$ and $\{x_n(w)\}$ be the random SP algorithm of three operators with errors defined as follows:*

$$\begin{aligned} x_{n+1}(w) &= (1 - \alpha_n)y_n(w) + \alpha_n T_1(w, y_n(w)) + u_n(w) \\ y_n(w) &= (1 - \beta_n)y_n(w) + \beta_n T_2(w, z_n(w)) + v_n(w) \\ z_n(w) &= (1 - \gamma_n)y_n(w) + \gamma_n T_3(w, x_n(w)) + w_n(w), \end{aligned} \qquad \text{(RSPTO)}$$

*where $\{u_n(w)\}$, $\{v_n(w)\}$, $\{w_n(w)\}$ are sequences of measurable mappings from $\Omega$ to $C$ with $\Sigma \int_\Omega u_n(w)\, d\mu(w) < \infty$, $\Sigma \int_\Omega v_n(w)\, d\mu(w) < \infty$, $\Sigma \int_\Omega w_n(w)\, d\mu(w) < \infty$ and $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$. Then the common random fixed point of the random operators $\{T_i, i = 1, 2, 3\}$ is Bochner integrable if and only if for all $w \in \Omega$, $\lim_{n\to\infty} \inf \int_\Omega d(x_n(w), CRF) d\mu(w) = 0$, provided*

$$\int_\Omega \|T_i(w, \xi(w)) - \xi(w)\| d\mu(w) = 0 \quad \textit{implies} \quad \|T_i(w, \xi(w)) - \xi(w)\| = 0.$$

*Proof.* Verbatim repetition of the proof of Theorem 3.11 and is omitted. □

## 5. Conclusions

We have studied Bochner integrability of random fixed point and almost sure stability with respect to random Kirk type algorithms of generalized $\phi$-weakly contractive operators on a separable Banach space. Our results include generalization, refinement and random version of some well-known results:

(1) Our Theorems 3.3 and 3.4 extend and generalize, respectively, Theorems 1 and 3 by Zhang et al. [34] and provide random version of Theorems 3, 4, 5, 10-11 by Rhoades [29] and many results given in the book of Berinde [5].

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(2) Our fixed point result, Theorem 3.11, corrects and sets analogue of Theorems 3.1 and 3.4 by Okeke and Abbas [27].
(3) A random analogue of Theorems 2.6 and 2.4 by Hussain et al. [12] is given in Theorems 3.4 and 4.1, respectively.
(4) Theorem 3.7 extends and provides random version of Theorems 9-10 by Gursoy and Karakaya [9].
(5) Stochastic generalization of Theorem 8 by Kumar et al. [23] is presented in Theorem 3.9.

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# सहृदय

(भाषा, साहित्य, संस्कृति, संवेदना और शोध का त्रैमासिक)

(केंद्रीय हिंदी निदेशालय, मानव संसाधन विकास मंत्रालय के
आंशिक वित्तीय सहयोग से प्रकाशित)

## मध्यकालीन हिंदी साहित्य : संवेदना, शास्त्र और समसामयिकता
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डॉ. विकास कुमार

# वृंद के काव्य में 'व्यवहार-दर्शन'

रीतिकाल की लगभग दो सौवर्ष पुरानी लंबी काव्य परंपरा में अनेक [illegible] कवियों ने जन्म लिया है और बिहारी, घनानंद जैसे कवियों ने तो अपनी [illegible] के बल पर हिंदी साहित्य के अग्रगण्य कवियों में अपना स्थान बनाया है। परंतु [illegible] में वृंद जैसे कवि भी हुए हैं जो बिहारी, घनानंद की भाँति उतने विख्यात तो [illegible] पर केवल इसी बात से उनकी बहुमुखी प्रतिभा को कम करके नहीं आँका जा [illegible]।

रीतिकाल बेशक शृंगार के प्राधान्य का काल है लेकिन रीतिकालीन [illegible] अपना एक महत्व है और रीतिकाल के नीति-काव्य रचयिताओं में वृंद का [illegible] है। डॉ. जनार्दन राव चेलेर ने आचार्य हजारी प्रसाद को उद्धृत करते हुए लिखा है [illegible] "इनकी सतसई के दोहे उत्तर मध्यकाल में बहुत सम्मान के साथ पढ़े व पढ़ाए [illegible] हैं।"[1] आचार्य रामचंद्र शुक्ल ने इन्हें अच्छा सूक्तिकार कहा है।[2] डॉ. रामस्वरूप [illegible] 'रसिकेश' ने इन्हें प्रमुख नीति कवि माना है[3] और डॉ. भोलानाथ तिवारी ने तो इनके [illegible] की तुलना महाकवि तुलसीदास के छंदों से की है।[4]

वृंद के काव्य का अनुशीलन करने पर अनेक ऐसे तथ्य दृष्टिगोचर होते है जो [illegible] को जीवन की विभिन्न परिस्थितियों में उचित व्यवहार की सीख देते हैं। वस्तुतः [illegible] काव्य को 'व्यवहार-दर्शन' की संज्ञा देना भी असमीचीन न होगा क्योंकि छोटी-छोटी [illegible] बातों से 'व्यवहार-ज्ञान' की बड़ी-बड़ी गंभीर दार्शनिकता उनके काव्य में झलकती है। [illegible] है –

नीकी पै फीकी लगै बिनु अवसर की बात।
जैसे बरनन युद्ध मैं रस सिंगार न सुहात।।[5]

अर्थात् कोई भी बात चाहे कितनी ही अच्छी क्यों न हो यदि बिना अवसर के [illegible] जाए तो व्यर्थ है। युद्ध के समय शृंगार की बात करना अच्छा नहीं लगता।

रोचक बात है कि फीकी (गलत) बात भी यदि समयानुकूल हो तो भली लग सकती है। गाली देना गलत माना जाता है। लेकिन विवाह के समय की गाली भी प्रसन्न [illegible] है –

फीकी पै नीकी लगै कहिए समय विचारि।
सब को मन हरषित करै ज्यों विवाह में गारि।।[6]

पानी में रहकर मगर से बैर नहीं किया जाता। इस उक्ति की पुष्टि वृंद भी करते हैं –

कैसे निबहै निबल जन, करि सबलन सौं गैर।
जैसे बस सागर विषै करत मगर सौं बैर।।[7]

अर्थात् कमजोर व्यक्ति बलवान को नाराज कर कैसे निभा सकता है? जैसे समुद्र में रहकर मगर से बैर करके रहना कठिन है।

कवि का स्पष्ट मत है कि किसी कार्य को हाथ में लेने से पूर्व अपनी शक्ति का अंदाज लगा लेना चाहिए। चादर जितनी लंबी हो, पाँव उतने ही पसारने चाहिए –

अपनी पहुँच विचारि कै करतब करियै दौर।
तेते पाँव पसारिये जेती लाँबी सौर।।[8]

वृंद सलाह देते हैं कि जैसा हवा का रूख हो वैसा ही व्यवहार करना चाहिए। बात बनती हो उसे बना लेना चाहिए –

बनती देख बनाइयै परन न दीजै खोट।
जैसी चलै बयार तब तैसी दीजै ओट।।[9]

कवि का मानना है कि बेमेल काम करने वाले की हँसी होती ही है। योगी अगर भोग की लालसा रखने लगे तो कौन उसे योगी कहेगा। द्रष्टव्य है –

अनमिलती जोई ताही कौ उपहास।
जैसे जोगी जोग में करत भोग की आस।।[10]

ध्यान देने योग्य तथ्य है कि वृंद बार-बार इस तथ्य को रेखांकित करते हैं कि किसी बड़े व्यक्ति का आश्रय पाने से, उसके साथ रहने से लाभ अवश्य होता है। वे तो यहाँ तक मानते हैं कि जीवन में मनुष्य की छोटाई या बड़ाई अर्थात् यश या अपयश, सम्मान या अपमान आदि उसके 'बड़े व्यक्तियों' के साथ संबंधों पर ही निर्भर करते हैं –

गुरुता लघुता पुरुष आश्रय ब तै होय।
करी बृंद मैं विंध्य सौं दर्पण मैं लघु सोय।।[11]

अर्थात् मनुष्य की गुरुता या लघुता (छोटई या बड़ाई) उसके आश्रय (आश्रयदाता) पर निर्भर करती है। हाथी समूह में चलने पर विंध्याचल पर्वत के समान जान पड़ता है और दर्पण में छोटा दिखता है। इसी प्रकार,

रहे समीप बड़ेन के होत बड़ो हित मेल।
सबही जानत बढ़त हैं वृक्ष बराबर बेल।।[12]

अर्थ यह है कि बड़े-बड़ों के साथ रहने से लाभ ही होता है। जैसे वृक्ष के साथ रहने

के कारण लता भी वृक्ष के बराबर ही बढ़ती है।

रोचक बात यह है कि 'बड़ों' के साथ मित्रता के लाभ का उल्लेख तो निस्संदेह अन्य नीति-कवि भी करते हैं। तुलना करें –

जोकर ऊँचा बैठना जेकर खेत निवान।
ओकर बैरी का करै जेकर मीत दीवान।।[13]

अर्थात् जो बड़े-बड़े लोगों में उठता-बैठता हो, दीवान स्वयं जिसका मित्र हो, उसका शत्रु भी क्या बिगाड़ सकता है? लेकिन वृंद ने यह अवश्य ध्यान रखने को कहा है कि आश्रय उसी का लेने में लाभ है जिससे कुछ मिल सकता हो। सूखे तालाब पर जाने से प्यास नहीं बुझा करती –

जाहीं तें कछु पाइयै करियै ताकी आस।
रीते सरवर पै गएँ कैसे बुझत प्यास।।[14]

और सबसे रोचक नियत जो कवि ने 'बड़े' लोगों के साथ के संबंध में उल्लिखित किया है, वह यह है कि बड़े जो कहें, वही करना चाहिए लेकिन जो करें वह नहीं करना चाहिए। नंगे रहने पर भी शिव महादेव कहलाते हैं, सामान्य जन ऐसा करे तो पागल कहलाएगा –

बड़े कहैं सो कीजियै करैं सु करियै नाँहि।
हर ज्यौं पंचन मैं फिरैं और जे बिकत कहाहिं।।[15]

सदा से ही माना जाता रहा है कि महत्वपूर्ण निर्णय करते समय बुद्धिमान लोगों से सलाह करना अच्छा रहता है। वृंद ने भी यही कहा है कि जहाँ सौ चतुरों की राय मिलती है वह राय पक्की (अच्छी) होती है।

सौं जु याने क मति यहै कहावत साँच।
कोंचहि पाच कहै न कोऊ पाचहि कहै न काँच।।[16]

और व्यवहार-ज्ञान के बारे में कवि की सीख यह भी है कि जो जैसा हो उसके साथ व्यवहार भी वैसा ही करना चाहिए। कठोर काठ को भौंरा छेद देता है लेकिन कोमल कमल में बंद हो जाता है। कठोर के साथ कठोर और कोमल के साथ कोमल व्यवहार ही समीचीन है –

जो जैसे तिहि तैसिये करियै नीति प्रकास।
काठ कठिन भेदै भ्रमर मृदु अरविंद निवास।।[17]

कुल मिलाकर हम देखते हैं कि सामान्य व्यवहार के विभिन्न पहलुओं को वृंद ने स्व-अनुभवों के आधार पर सरलतम भाषा में प्रस्तुत किया है। कवि अपनी बात को समझाने के लिए सुंदर दृष्टांत साथ-साथ देते चलते हैं। उन्होंने जीवन के व्यवहारिक पक्ष की सूक्ष्मता से परख की और उस परख को अपने काव्य में उकेरा। ये सैद्धांतिक ज्ञान नहीं

व्यावहारिक ज्ञान का दर्शन-शास्त्र बड़ी सरलता से समझाते हैं। अपनी सामर्थ्य को समझकर काम करने की सीख इन्होंने दी है तो अवसरानुसार बातचीत का निर्देश भी दिया है। शक्तिशाली से बिगाड़ना अच्छा नहीं है तो 'बड़ों का आश्रय पाने में लाभ ही है — ऐसा उनका विश्वास है, बेशक सैद्धांतिक कसौटी पर यह चाटुकारिता की श्रेणी में ही क्यों न रखा जा सकता हो। आश्रयदाता कुछ देने की स्थिति में भी है या नहीं — इसको ठोक-बजा कर परखने का उपदेश भी कवि देते हैं। क्योंकि सूखे तालाब पर जाने से प्यास नहीं बुझा करती। जीवन के बड़े निर्णय अनुभवी बुद्धिमान लोगों से सलाह-मशविरा करके करने में ही समझदारी होती है और कवि के व्यवहार-ज्ञान का सार 'बड़े कहें सो कीजिए', वाली उक्ति में लक्षित किया जा सकता है। उनकी 'नीति सतसई', 'व्यवहार-दर्शन' का अगाध भंडार है।

□

संदर्भ


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# Stresses in an Orthotropic Elastic Layer Lying Over an Irregular Isotropic Elastic Half-Space

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## ABSTRACT

**Objective:** The objective is to obtain the stresses due to strip loading in orthotropic plate lying over an irregular isotropic elastic medium.

**Methods:** Anti-plane strain problem with perfect bonding boundary conditions following by Fourier Transformation on the equilibrium equation are used to obtain the solution.

The deformation field due to shear line load at any point of the medium consisting of an orthotropic elastic layer lying over an irregular isotropic elastic medium is obtained. The anti-plane strain problem with the presence of rectangular irregularity is considered. In order to study the effect of irregularity present in the medium and of anisotropy of the layer, we computed shearing stresses in both the media graphically.

**Key Words:** Orthotropic, Shear load, Anti-plane strain, Rectangular irregularity

## INTRODUCTION

It is well known that the upper part of the Earth is recognized as having orthorhombic symmetry. Orthorhombic Symmetry is also expected to occur in sedimentary basins as a result of combination of vertical cracks with a horizontal axis of symmetry and periodic thin layer anisotropic with a vertical symmetry axis. When one of the planes of symmetry in an orthorhombic symmetry is horizontal, the symmetry is termed as orthotropic symmetry and most symmetry systems in the Earth crust also have orthotropic orientations (Crampin[1]).

The problem of deformation of a horizontally layered elastic material due to surface loads is of great interest in geosciences and engineering. In material science engineering, the applications related to laminate composite material are increasing. Many works related to Earth, such as fills or pavements consist of layered elastic medium. When the source surface is very long, then a two-dimensional approximation simplifies the algebra and one can easily obtain a closed form analytical solution. The deformation field around mining tremors and drilling into the crust of the Earth can be analyzed by the deformation at any point of the media due to strip-loading. It also contributes for theoretical consideration of volcanic and seismic sources as it account for the deformation fields in the entire medium surrounding the source region. It may also find application in various engineering problems regarding the deformation of layered isotropic and anisotropic elastic medium (Garg *et al*[2], Singh *et al*[3]).

The study of static deformation with irregularity present in the elastic medium due to continental margin, mountain roots etc is very important to study. Chattopadhyay[4], Kar *et al*[5], De Noyer[6], Mal[7], Acharya and Roy[8] discussed the problems with irregular thickness. Love[9] provided the solution of static deformation due to line source in an isotropic elastic medium. Salim[10] studied the effect of rectangular irregularity on the static deformation of initially stressed and unstressed isotropic elastic medium respectively. The distribution of the stresses due to strip-loading in a regular monoclinic elastic medium had been studied by Madan *et al*[11]. The effect of rigidity and irregularity present in fluid-saturated porous anisotropic single layered and multilayered elastic media on the propagation of Love waves had been analyzed by Madan *et al*[12] and Kumar *et al*[13] respectively. Recently, Madan and Gabba[14] studied two-dimensional deformation of an irregular orthotropic elastic medium due to normal line load.

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In this paper, we have obtained the closed-form expressions for the displacement and shearing stresses in a horizontal orthotropic elastic plate of an infinite lateral extent lying over an irregular isotropic base due to strip-loading. Numerically, at different sizes of irregularity, we have studied the variations of shearing stresses with horizontal distance and it has been observed that the shearing stresses show significant variation with horizontal distance at the different depth levels.

## PROBLEM FORMULATION

Consider a horizontal orthotropic elastic plate of thickness H lying over an infinite isotropic elastic medium with $x_1$–axis vertically downwards. The origin of the Cartesian coordinate system $(x_1, x_2, x_3)$ is taken at the upper boundary of the plate. The orthotropic elastic plate occupies the region $0 \le x_1 \le H$ and is described as Medium I whereas the region $x_1 > H$ is the isotropic elastic half space over which the plate is lying and is described as Medium II. (Fig. 1)

Suppose a shear load $P$ per unit area is acting over the strip $|x_2| \le h$ of the surface $x_1 = 0$ in the positive $x_3$–direction. The boundary condition at the surface $x_1 = 0$ is

$$\tau_{31} = \begin{cases} -P \mid x_2 \mid \le h \\ 0 \mid x_2 \mid > h \end{cases} \quad (1)$$

The irregularity is assumed to be rectangular with length $2a$ and depth $d$. The equation of the rectangular irregularity may be represented as

$$x_1 = \varepsilon f(x_2) = \begin{cases} d \mid x_2 \mid \le a \\ 0 \mid x_2 \mid > a \end{cases} \quad (2)$$

where $\varepsilon = \frac{d}{2a} << 1$ is the perturbation factor.

## THEORY

The equilibrium equations of Cartesian coordinate system $(x_1, x_2, x_3)$ for zero body force are

$$\sigma_{1,1} + \tau_{12,2} + \tau_{13,3} = 0 \quad (3)$$

$$\tau_{21,1} + \sigma_{2,2} + \tau_{23,3} = 0 \quad (4)$$

$$\tau_{31,1} + \tau_{32,3} + \sigma_{3,3} = 0 \quad (5)$$

where $\sigma_1, \sigma_2, \sigma_3$ are normal stresses and $\tau_{12}, \tau_{13}, \tau_{21}, \tau_{23}, \tau_{31}, \tau_{32}$ are called shearing stresses.

The stress-strain relations for- an orthotropic elastic medium with co-ordinate planes as planes of elastic symmetry are

$$\left.\begin{aligned} \sigma_1 &= C_{11}e_1 + C_{12}e_2 + C_{13}e_3 \\ \sigma_2 &= C_{21}e_1 + C_{22}e_2 + C_{23}e_3 \\ \sigma_3 &= C_{13}e_1 + C_{23}e_2 + C_{33}e_3 \\ \tau_{23} &= 2C_{44}e_{23} \\ \tau_{13} &= 2C_{55}e_{13} \\ \tau_{12} &= 2C_{66}e_{12} \end{aligned}\right] \quad (6)$$

where $e_1, e_2, e_3$ are normal strain components and $e_{12}, e_{23}, e_{13}$ are normal strain components. The suffices $C_{ij}$ $(i, j = 1,2,3,4,5,6)$ are stiffnesses of an orthotropic elastic material.

The strain - displacement relations are given as

$$e_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) \quad \text{and} \quad e_1 = \frac{\partial u_1}{\partial x_1}, \text{etc.} \quad (7)$$

In terms of displacement components, the equilibrium equations can be written from equations (3) – (7) as :

$$C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + C_{55}\frac{\partial^2 u_1}{\partial x_3^2} + (C_{12} + C_{66})\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + (C_{13} + C_{55})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} = 0 \quad (8)$$

$$_6 + C_{12})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2} + C_{44}\frac{\partial^2 u_2}{\partial x_3^2}(C_{23} + C_{44})\frac{\partial^2 u_3}{\partial x_3 \partial x_2} = 0 \quad (9)$$

$$\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + (C_{44} + C_{23})\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + C_{55}\frac{\partial^2 u_3}{\partial x_1^2} + C_{44}\frac{\partial^2 u_3}{\partial x_2^2} + C_{33}\frac{\partial^2 u_3}{\partial x_3^2} = 0 \quad (10)$$

Consider the field equation of an orthotropic material in anti – plane strain equilibrium state as:

$$u_1 = u_2 = 0, \quad u_3 = u_3(x_1, x_2); \quad (11)$$

The non-zero stresses for an anti – plane strain equilibrium state are

$$\tau_{31} = C_{55}\frac{\partial u_3}{\partial x_1} \quad (12)$$

$$\tau_{32} = C_{44}\frac{\partial u_3}{\partial x_2} \quad (13)$$

Equilibrium Equations for an orthotropic elastic medium due to anti – plane strain deformation are found to be

$$\frac{\partial^2 u_3}{\partial x_1^2} + m^2\frac{\partial^2 u_3}{\partial x_2^2} = 0 \quad (14)$$

where $m = \sqrt{\frac{C_{44}}{C_{55}}}$.

At the interface $(y, x = \varepsilon f(y))$, the boundary conditions are:

1. $u_3^I = u_3^{II}$.
2. $\tau_{31}^I - i\varepsilon f'(y)\tau_{32}^I = \tau_{31}^{II} - i\varepsilon f'(y)\tau_{32}^{II}$. (15)

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By using the boundary condition (15), we find the deformation field at any point of the orthotropic elastic plate corresponding to irregular contact between the orthotropic plate and the isotropic elastic half space due to strip-loading.

Taking the Fourier transform of the equilibrium equation (14), we get

$$\frac{d^2\bar{u}_3^I}{dx_1^2} - 2\left(\frac{C_{45}}{C_{55}} ik\right)\frac{d\bar{u}_3^I}{dx_1} - \frac{C_{44}}{C_{55}} k^2 \bar{u}_3^I = 0 \qquad (16)$$

The solution of the ordinary differential equation is

$$\bar{u}_3^I = \left(C_1 e^{m|k|x_1} + C_2 e^{-m|k|x_1}\right) \qquad (17)$$

where $C_1$ and $C_2$ may be functions of $k$

By using inverse Fourier transform, we have

$$u_3^I = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(C_1 e^{m|k|x_1} + C_2 e^{-m|k|x_1}\right)e^{-ix_2k}\,dk \qquad (18)$$

Using equation (12), (13) and (18), the shear stresses are

$$\tau_{31}^I = \frac{T_1}{2\pi}\int_{-\infty}^{\infty}\left(C_1 e^{m|k|x_1} - C_2 e^{-m|k|x_1}\right)e^{-ix_2k}|k|\,dk \qquad (19)$$

$$\tau_{32}^I = \frac{T_1}{2\pi}\left[-im\int_{-\infty}^{\infty}\left(C_1 e^{m|k|x_1} + C_2 e^{-m|k|x_1}\right)e^{-ix_2k}k\,dk\right] \qquad (20)$$

Where $T_1 = mC_{55} = \sqrt{C_{11}C_{55}}$ U. Using the boundary condition (1), we have

$$\bar{\tau}_{31}^I = -\frac{2P}{\pi}\sin kh \qquad (21)$$

Therefore

$$\tau_{31}^I = -\frac{P}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin kh}{k}\right)e^{-ikx_2}\,dk \qquad (22)$$

From (19) and (21), we obtain

$$C_1 - C_2 = -\frac{2P}{T_1}\left(\frac{\sin kh}{k|k|}\right) \qquad (23)$$

The displacement in the isotropic elastic half space $x_1 > H$ is

$$u_3^{II} = \frac{1}{2\pi}\int_{-\infty}^{\infty} C_2' e^{-|k|x_1} e^{-ix_2k}\,dk \qquad (24)$$

The coefficient $C_2'$ is to be determined from the boundary conditions.

From equations (12), (13) and (17), we obtain

$$\tau_{31}^{II} = -\frac{\mu}{2\pi}\int_{-\infty}^{\infty} C_2' e^{-|k|x_1} e^{-ix_2k}|k|\,dk \qquad (25)$$

$$\tau_{32}^{II} = -\frac{i\mu}{2\pi}\int_{-\infty}^{\infty} C_2' e^{-|k|x_1} e^{-ix_2k}k\,dk \qquad (26)$$

Equations (15), (18), (19), (20), (24), (25) and (26) yield the relation

$$C_1 e^{\varepsilon m|k|f(y)} + C_2 e^{-\varepsilon m|k|f(y)} = C_2' e^{-\varepsilon|k|f(y)} \qquad (27)$$

$$T\left(k' - m\varepsilon f'(y)\right)C_1 e^{\varepsilon m|k|f(y)} - T\left(k' + m\varepsilon f'(y)\right)C_2 e^{-\varepsilon m|k|f(y)} + \left(k' + \varepsilon f'(y)\right)C_2' e^{-\varepsilon|k|f(y)} \qquad (28)$$

where $T = \frac{T_1}{\mu}$ and $k' = \frac{|k|}{k}$.

Solving (23), (27) and (28), we get

$$C_1 = \frac{2P\sin kh}{T_1 k|k|}\left(\frac{(k' + \varepsilon f'(y)V')e^{-2\varepsilon m|k|f(y)}}{k'\left(V - e^{-2\varepsilon m|k|f(y)}\right) - \varepsilon f'(y)V'\left(1 + e^{-2\varepsilon m|k|f(y)}\right)}\right) \qquad (29)$$

$$C_2 = \frac{2P\sin kh}{T_1 k|k|}\left(1 + \frac{(k' + \varepsilon f'(y)V')e^{-2\varepsilon m|k|f(y)}}{k'\left(V - e^{-2\varepsilon m|k|f(y)}\right) - \varepsilon f'(y)V'\left(1 + e^{-2\varepsilon m|k|f(y)}\right)}\right) \qquad (30)$$

$$C_2' = \frac{2P\sin kh}{T_1 k|k|}\left(\frac{k'(1+V)}{k'\left(V - e^{-2\varepsilon m|k|f(y)}\right) - \varepsilon f'(y)V'\left(1 + e^{-2\varepsilon m|k|f(y)}\right)}\right)e^{-\varepsilon(m-1)|k|f(y)} \qquad (31)$$

where $V = (T-1)/(T+1)$.

## PROBLEM SOLUTION

By applying Fourier Transformation technique on equation (2) we obtained

$$\bar{f}(k) = \frac{4a}{k}\sin(ka) \qquad (32)$$

Therefore, by using inverse transformation, we have

$$f(y) = \frac{1}{2\pi}\sin(ka)\,e^{-iky}\,dk = sign(a - x_2) + sign(a + x_2) \qquad (33)$$

where is the signum function.

By substituting the values of constants $C_1, C_2$ and $C_1'$ from equations (29), (30), (31) in the equations (18), (19), (20) for Medium I and in (24), (25), (26) for Medium II and also, substituting the value of $f(y)$ for rectangular irregularity from equation (33), we obtain the following expressions for displacement and stresses.

For Med. I

$$u_3^I = \frac{P}{\pi T_1}\int_{-\infty}^{\infty}\frac{\sin kh}{k|k|}\left\{\left(1 + \sum_{n=1}^{\infty} V^n e^{m|k|\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)\right)}\right)\left(e^{m|k|x_1} + Ve^{-m|k|x_1}\right)\right\}e^{-ikx_2} \qquad (34)$$

$$\tau_{31}^I = \frac{P}{\pi}\Bigg[(1+V)\tan^{-1}\left(\frac{2hmx_1}{x_2^2 + m^2x_1^2 - h^2}\right) + \sum_{n=1}^{\infty} V^n\Bigg\{\tan^{-1}\left(\frac{2hm\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)}{x_2^2 + m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)^2 - h^2}\right) - V\tan^{-1}\left(\frac{2hm\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)}{x_2^2 + m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2 - h^2}\right)\Bigg\}\Bigg] \qquad (35)$$

$$\tau_{32}^I = -\frac{Pm}{4\pi}\Bigg[(1+V)\log\frac{(h+x_2)^2 + m^2x_1^2}{(h-x_2)^2 + m^2x_1^2} - \sum_{n=1}^{\infty} V^n\Bigg\{\log\frac{(h+x_2)^2 + m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)^2}{(h-x_2)^2 + m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)^2} + V\log\frac{(h+x_2)^2 + m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2}{(h-x_2)^2 + m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2}\Bigg\}\Bigg] \qquad (36)$$

For Med. II

$$u_3^{II} = -\frac{P}{\pi T_1}\int_{-\infty}^{\infty}\frac{\sin kh}{k|k|}(1+V)\left(1 + \sum_{n=1}^{\infty} V^n e^{2nm\varepsilon|k|(sign(a-x_2)+sign(a+x_2))}\right)e^{|k|\left((m+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)}e^{-ikx_2}dk \quad (37)$$

$$\tau_{31}^{II} = -\frac{P\mu}{\pi T_1}(1+V)\left[\tan^{-1}\left(\frac{2h\left((m+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)}{x_2^2+\left((m+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)^2-h^2}\right) + \sum_{n=1}^{\infty}V^n\left\{\tan^{-1}\left(\frac{2h\left((2m(n+1)+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)}{x_2^2+\left((2m(n+1)+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)^2-h^2}\right)\right\}\right] \quad (38)$$

$$\tau_{32}^{II} = \frac{P\mu}{2\pi T_1}(1+V)\left[\log\frac{(h+x_2)^2+\left((m+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)^2}{(h-x_2)^2+\left((m+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)^2} + \sum_{n=1}^{\infty}V^n\log\frac{(h+x_2)^2+\left((2m(n+1)+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)^2}{(h-x_2)^2+\left((2m(n+1)+1)\varepsilon(sign(a-x_2)+sign(a+x_2))-x_1\right)^2}\right] \quad (39)$$

## NUMERICAL RESULTS AND DISCUSSION

In this section, we intend to examine the effect of irregularity on the stresses due to shear line load acting at any point of the orthotropic elastic layer lying over an irregular isotropic half space. For numerical computation, we use the values of elastic constants of Topaz (Orthotropic) for Medium I and the values of elastic constants of Glass (Isotropic) for Medium II given by Love[9].

Figures (2)-(4) and Figures (5)-(7) show the variation of shearing stresses $\tau_{31}^{I}$ and $\tau_{32}^{I}$ respectively, with horizontal distancefor different values of $a = 1, 1.2, 1.4, 1.6$ and for different depth levels $x_1 = 0.5, 1, 1.5$. Figures (5)-(7) clearly show that for different values of, the difference between shearing stresses in magnitude significantly decreases as the depth increases.

Figures (8)-(10) and Figures (11)-(13) show the variation of shearing $\tau_{31}^{II}$ and $\tau_{32}^{II}$ respectively with horizontal distancefor $x_2$ different values of $a = 1, 1.2, 1.4, 1.6$. It has been found from the Figures (8)-(10) that for different values of $a$, the difference between shearing stresses in $\tau_{31}^{II}$ magnitude significantly increases as the depth increases.

## CONCLUSIONS

The explicit expressions for the shearing stresses in an elastic medium consisting of orthotropic elastic layer lying over an irregular isotropic half space due to shear loading has been obtained. The results obtained are useful to study the static deformation around mining tremors and drilling into the crust of the Earth. The results are also useful to study the effect of irregularity present between the layer and the half-space. Graphically, it has been observed that the difference between the shearing stresses in magnitude in orthotropic elastic layer decreases as depth increases due to irregularity present.

Further, it has also been observed that in isotropic semi-infinite half-space, the difference between the stresses in magnitude increases with the increase of depth. Thus, it has been concluded that the stress distribution in a layer with irregularity present at the interface is affected by not only the presence of irregularity but also by anisotropy of the elastic medium as a result of shear load acting over the strip of an orthotropic elastic medium.

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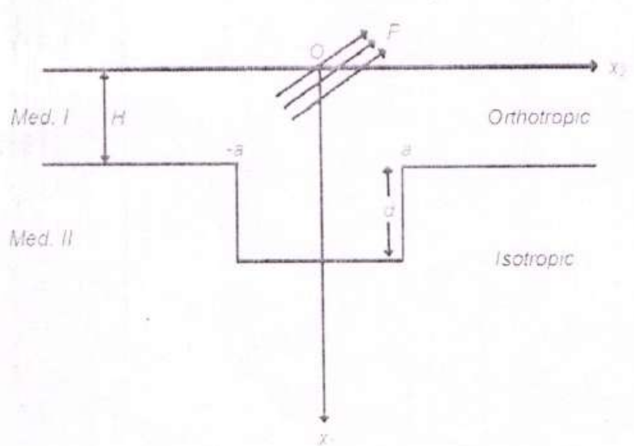


Figure 1: Section of the Model.

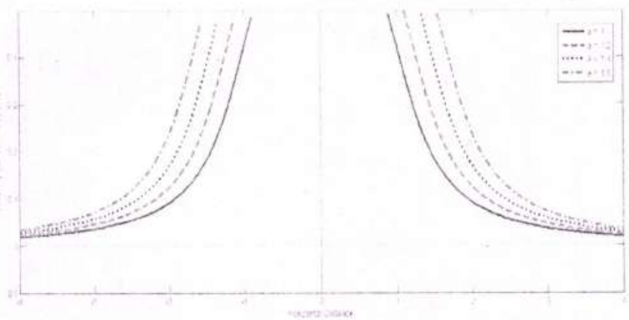

Figure 2: Variation of the Shearing Stress $\tau^{I}_{31}$ in Med. I with the horizontal distance $x_2$ at $x_1$= 1.

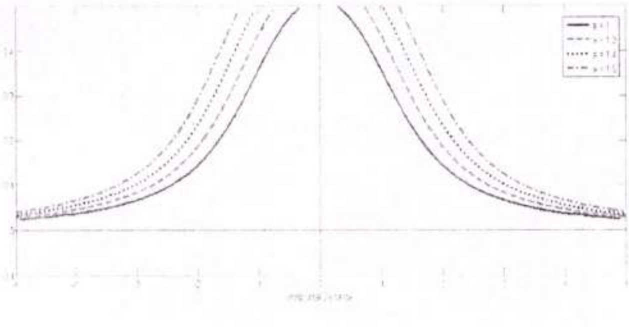

Figure 3: Variation of the Shearing Stress $\tau^{I}_{31}$ in Med. I with the horizontal distance $x_2$ at $x_1$= 1.

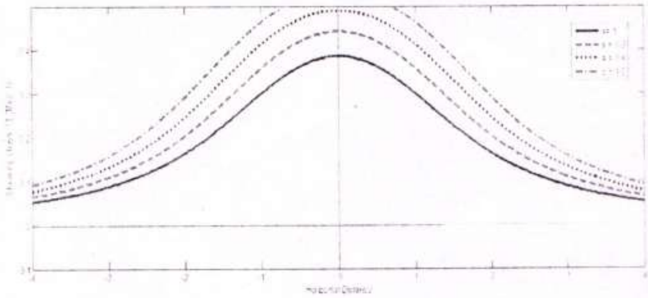

Figure 4: Variation of the Shearing Stress $\tau^{I}_{31}$ in Med. I with the horizontal distance $x_2$ at $x_1$= 1.5.

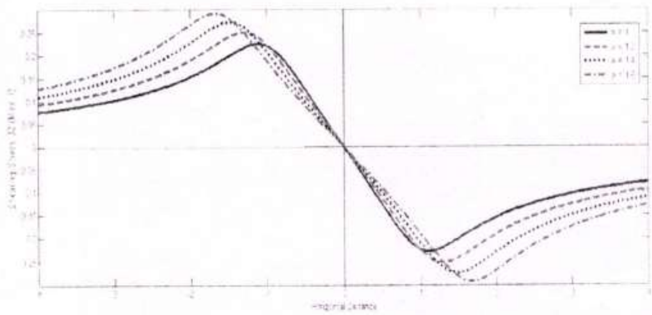

Figure 5: Variation of the Shearing Stress $\tau^{I}_{32}$ in Med. I with the horizontal distance $x_2$ at $x_1$= 0.5.

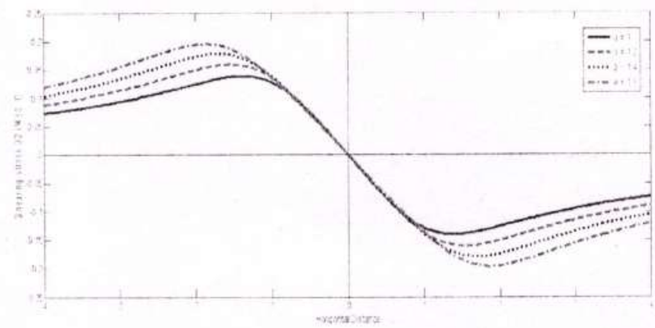

Figure 6: Variation of the Shearing Stress $\tau^{I}_{32}$ in Med. I with the horizontal distance $x_2$ at $x_1$= 1.

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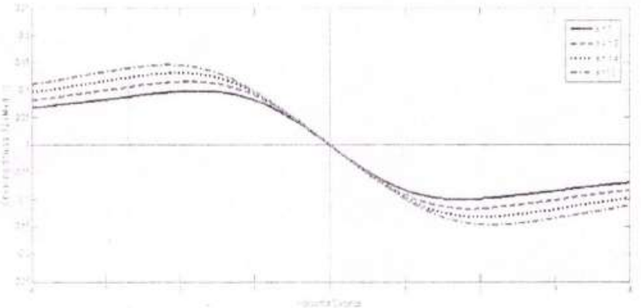

Figure 7: Variation of the Shearing Stress $\tau'_{32}$ in Med. I with the horizontal distance $x_2$ at $x_1 = 1.5$.

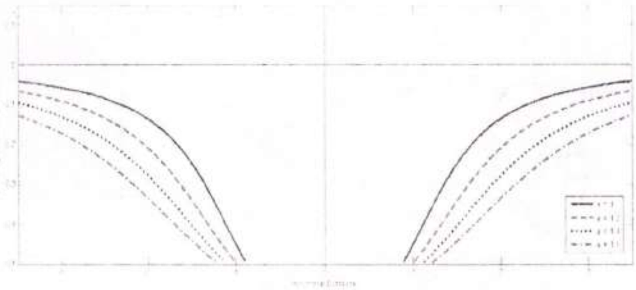

Figure 8: Variation of the Shearing Stress $\tau''_{31}$ in Med. II with the horizontal distance $x_2$ at $x_1 = 0.5$.

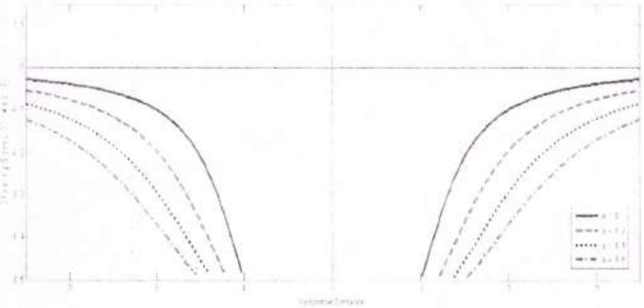

Figure 9: Variation of the Shearing Stress $\tau''_{31}$ in Med. II with the horizontal distance $x_2$ at $x_1 = 1$.

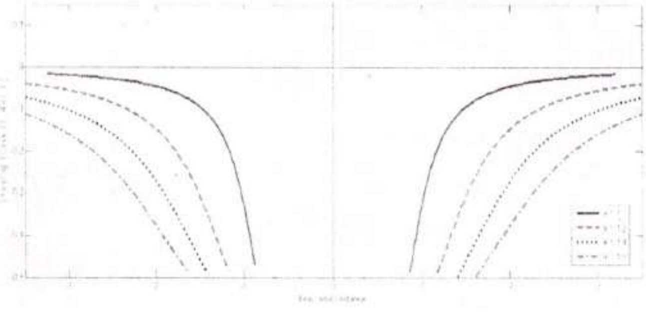

Figure 10: Variation of the Shearing Stress $\tau''_{31}$ in Med. II with the horizontal distance $x_2$ at $x_1 = 0.5$.

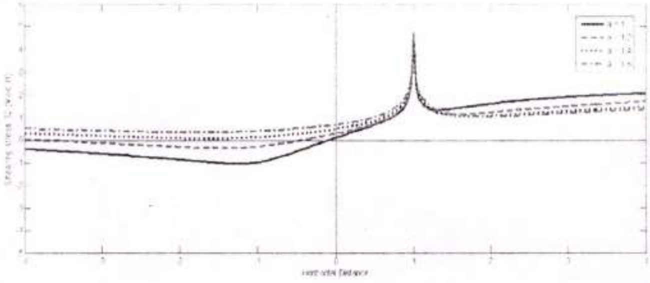

Figure 11: Variation of the Shearing Stress $\tau''_{32}$ in Med. II with the horizontal distance $x_2$ at $x_1 = 0.5$.

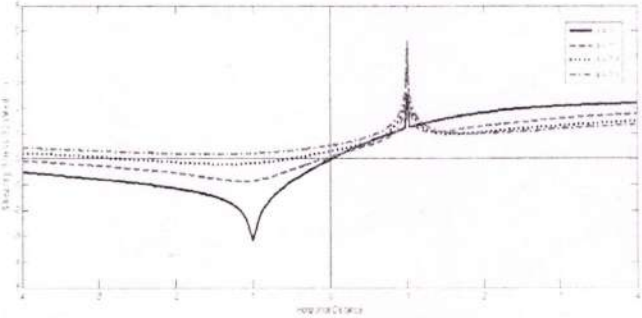

Figure 12: Variation of the Shearing Stress $\tau''_{32}$ in Med. II with the horizontal distance $x_2$ at $x_1 = 1$.

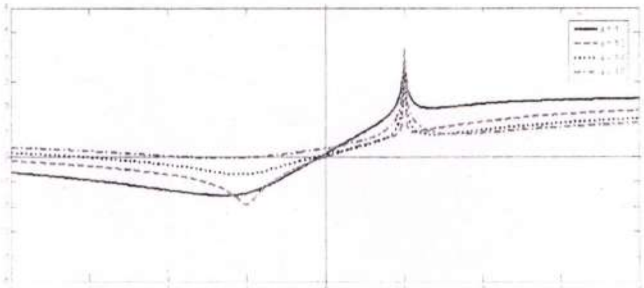

Figure 13: Variation of the Shearing Stress $\tau''_{32}$ in Med. II with the horizontal distance $x_2$ at $x_1 = 1.5$.

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# Geological Non-Uniform Strike-Slip Fault Model to Study Crustal Deformation in Monoclinic Elastic Medium with Width-Depth Ratio (WDR) of the Fault

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ABSTRACT

*The fault-width and fault-depth of the geological fault are the important parameters for the study of earthquakes. The width and depth of the fault influence the deformation field remarkably. The geometry of the fault plays a vital role in the study of crustal deformation. In this paper, a strike-slip dislocation model with exponential slip situated in a monoclinic elastic media is considered to study the effect of the width-depth ratio (WDR) on the static field. Both the uniform monoclinic semi-infinite elastic medium and two perfectly bonded semi-infinite anisotropic elastic media are considered. Graphically, for different values of $W_D(WDR)$, the variations of displacement with horizontal distance from the fault have been depicted*

**Keywords**: Exponential, Strike-slip, Fault-width, Fault-depth, Monoclinic, WDR.

**Mathematics Subject Classification**:74E1, 86A15

**Journal of Economic Literature(JEL) Classification**:C69

## 1. INTRODUCTION

The deformation of an isotropic and anisotropic elastic medium due to dislocation (uniform) has been studied by many researchers (Press 1965, Garg *et al* 1996, Rybicki 1971, Sharma and Garg 1993, Singh and Garg 1985). Kumar *et al* (2002) and Singh *et al* (2003) obtained expressions for the displacements at any point of the two-phase monoclinic elastic media and semi-infinite monoclinic elastic medium respectively due to uniform strike-slip dislocation. Madan *et al* (2005) obtained expressions to study the static deformation field in an orthotropic elastic semi-infinite medium due to non-uniform slip on a long vertical strike-slip fault.

Chugh *et al* (2010) obtained expressions for the deformation at any point of two-phase medium consisting of homogeneous orthotropic elastic semi-infinite medium in welded contact with homogeneous isotropic elastic semi-infinite medium caused by non-uniform slip. Kumar *et al* (2015) studied the fluid saturated porous elastic layer over a semi-infinite non-homogeneous elastic medium with a rectangular irregularity. Recently, Madan *et al* (2015) derived closed form analytical expression



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for displacement in monoclinic semi-infinite elastic medium due to exponential discontinuity along a very long strike-slip fault which varies along the depth by keeping width constant. Godara *et al* (2017) studied the static deformation due to non-uniform strike-slip fault in an orthotropic semi-infinite medium with rigid surface.

Till now, mostly the study of deformation field due to strike-slip fault with non-uniform slip has been done by taking either the width of the fault or the depth of the fault constant. In this paper, we have considered the variations of fault with the width as well as the depth of the vertical exponential strike-slip fault situated in monoclinic elastic semi-infinite medium and two perfectly bonded monoclinic elastic semi-infinite media. The results of Kumar *et al* (2002) and Singh *et al* (2003) for uniform slip models along vertical strike slip fault can be derived from the results obtained in the present paper. Numerically, the variation of dimensionless displacement in both the two models for different values of $W_D$ (width-depth) with dimensionless horizontal distance from the fault has been examined.

## 2. FORMULATION AND SOLUTION OF THE PROBLEM

### 2.1 Model I (Monoclinic Semi-infinite Elastic Medium)

Firstly, we take a monoclinic semi-infinite elastic half medium occupying the region $x_2 \geq 0$ and the $x_2-$axis vertically downward. We suppose that the strike of the fault is along the $x_3-$axis and let there be a vertical strike-slip fault of infinite length $-\infty < x_3 < \infty$ and finite width $W$ with non-uniform slip of exponential type along the fault situated on $x_2-$axis. Let $D$ be the fault depth and $W \leq D$ (Figure 2.1)

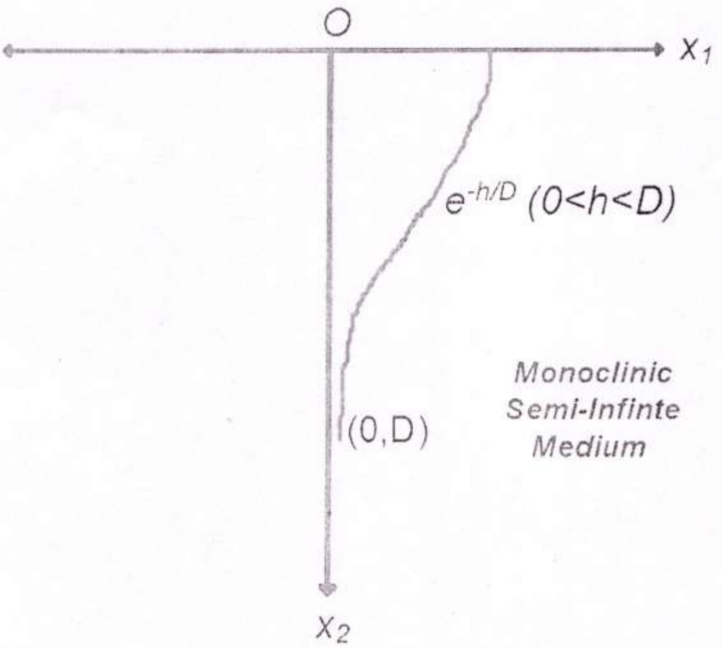


Fig. 1 Geometry of the fault for monoclinic semi-infinite medium (Exponential Discontinuity).

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Following Maruyama (1966), at any point of the monoclinic elastic half-space due to long strike-slip fault with $n_k$ as the unit normal to the fault section, the displacement field in terms of Green's function $G_{3k}^3(x,\xi)$ is given by

$$\omega(x) = \int \Delta\omega(\xi)\, G_{3k}^3(x,\xi) n_k(\xi) ds(\xi) \qquad (1)$$

where $\Delta\omega(\xi)$ is the displacement discontinuity and

$$G_{3k}^3(x,\xi) = C_{3k3s}\frac{\partial}{\partial \xi_s} G_3^3(x,\xi) \qquad (2)$$

For monoclinic elastic half-space

$$G_3^3 = -\frac{1}{2\pi M}\mathrm{Re}\ln[\{x_1-\xi_1+q(x_2-\xi_2)\}\{x_1-\xi_1+qx_2-\bar{q}\xi_2\}] \qquad (3)$$

In terms of polar co-ordinates, for fault-width $L$ and of infinite length along the strike direction:

$$\xi_1 = h\cos\delta, \quad \xi_2 = h\sin\delta$$

$$n_1 = -\sin\delta, \qquad n_2 = \cos\delta$$

(4)

Equations (1) – (2) and voight notation for stiffnesses

$$11 \to 1, 22 \to 2, 33 \to 3, 23 \to 4, 13 \to 5, 12 \to 6 \qquad (5)$$

give

$$\omega(x) = -\frac{1}{2\pi M}\int_L b\Big[(n_1C_{55}+n_2C_{45})\{x_1-\xi_1+q'(x_2-\xi_2)\}\left(\tfrac{1}{R^2}+\tfrac{1}{S^2}\right)+(n_1C_{45}+n_2C_{44})\Big\{q'\Big(x_1-\xi_1+ q'(x_2-\xi_2)\Big)\left(\tfrac{1}{R^2}+\tfrac{1}{S^2}\right)+q'^2\left(\tfrac{x_2-\xi_2}{R^2}-\tfrac{x_2+\xi_2}{S^2}\right)\Big\}\Big]ds \qquad (6)$$

where $b = \Delta\omega(\xi)$ is displacement discontinuity and

$$R^2 = \frac{1}{A+\varepsilon\sin 2\delta}\{[(A+\varepsilon\sin 2\delta)s - Cx_1 - B(x_2-d)]^2 + \lambda^2 Y_5^2\} \qquad (7)$$

$$S^2 = \frac{1}{A+\varepsilon\sin 2\delta}\{[(A+\varepsilon\sin 2\delta)s - C(x_1+2\varepsilon x_2) + B(x_2+d)]^2 + \lambda^2 Y_6^2\}$$

$$Y_5 = -x_1\sin\delta + (x_2-d)\cos\delta \qquad (8)$$

$$Y_6 = (x_1+2\varepsilon x_2)\sin\delta + (x_2-d)\cos\delta \qquad (9)$$

$$A = \cos^2\delta + \gamma\sin^2\delta \qquad (10)$$

$$B = \varepsilon\cos\delta + \gamma\sin\delta \qquad (11)$$

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$$C = \cos\delta + \varepsilon \sin\delta \tag{12}$$

$$q^{\cdot} = \varepsilon = -{C_{45}}/{C_{44}}, \quad \gamma = {C_{55}}/{C_{44}} \tag{13}$$

$$q^{\cdot\cdot} = \lambda = (\gamma - \varepsilon^2)^{1/2} = {M}/{C_{44}} \tag{14}$$

From figure 1, we have, for exponential variable slip along the fault

$$b = b(h) = b_0 \exp(-{h}/{D}), \qquad 0 < h < W \leq D, \tag{15}$$

where $b_0$ is the surface slip . As in the present case the slip is exponential variable slip ($\delta = 90°$) so that the point Q($\xi_1, \xi_2$) on the fault given by equation (4), reduces to

$$\xi_1 = 0, \quad \xi_2 = h$$

$$n_1 = 0, \; n_2 = 1 \tag{16}$$

Equations (7 – 12) yield

$$R^2 = \frac{1}{A}\{[Ah - Cx_1 - Bx_2]^2 + \lambda^2 x_1^2\} \tag{17}$$

$$S^2 = \frac{1}{A}\{[Ah - C(x_1 + 2\varepsilon x_2) + Bx_2]^2 + \lambda^2(x_1 + 2\varepsilon x_2)^2\} \tag{!8}$$

$$Y_5 = -x_1 \tag{19}$$

$$Y_6 = x_1 + 2\varepsilon x_2 \tag{20}$$

$$A = B = \gamma, \quad C = \varepsilon \tag{21}$$

By using equations (17 – 21), equation (6) simplifies to

$$\omega(x) = \frac{\lambda}{2\pi}\int_L \frac{(-x_1)b_0 \exp(-{h}/{D})}{\frac{1}{\gamma}\{(\gamma h - \varepsilon x_1 - \gamma x_2)^2 + \lambda^2 x_1^2\}} dh$$
$$- \frac{\lambda}{2\pi}\int_L \frac{(x_1 + 2\varepsilon x_2)b_0 \exp(-{h}/{D})}{\frac{1}{\gamma}\{(\gamma h - \varepsilon(x_1 + 2\varepsilon x_2) + \gamma x_2)^2 + \lambda^2(x_1 + 2\varepsilon x_2)^2\}} dh \tag{22}$$

We expand $\exp(-{h}/{D})$ by infinite Taylor series and since $\frac{h}{D} \ll 1$, therefore we may neglect the third and higher powers in the expansion of $\exp(-{h}/{D})$. Thus

$$\exp(-{h}/{D}) = 1 - \frac{h}{D} + \frac{h^2}{2D^2} \tag{23}$$

Substituting (23)in equation (22), we get



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$$\omega(x) = \frac{\lambda}{2\pi}\int_L \frac{(-x_1)b_0\left(1-\frac{h}{D}+\frac{h^2}{2D^2}\right)}{\frac{1}{\gamma}\{(\gamma h-\varepsilon x_1-\gamma x_2)^2+\lambda^2x_1^2\}}dh$$

$$-\frac{\lambda}{2\pi}\int_L \frac{(x_1+2\varepsilon x_2)b_0\left(1-\frac{h}{D}+\frac{h^2}{2D^2}\right)}{\frac{1}{\gamma}\{(\gamma h-\varepsilon(x_1+2\varepsilon x_2)+\gamma x_2)^2+\lambda^2(x_1+2\varepsilon x_2)^2\}}dh \quad (24)$$

Integrating using Wolfram Mathematica, the closed-form expression for the displacement varying with fault width(W) is obtained as

$$\omega(x) = \omega(h)|_0^W \quad (25)$$

where

$$\omega(h) = -\frac{b_0}{2\pi}\left[\tan^{-1}\left(\frac{\gamma h-\gamma x_2-\varepsilon x_1}{\lambda x_1}\right)+\tan^{-1}\left(\frac{\gamma h+\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)}{\lambda(x_1+2\varepsilon x_2)}\right)\right]$$
$$+\frac{\lambda b_0}{4\pi D\gamma}\Big[x_1\log((\gamma h-\gamma x_2-\varepsilon x_1)^2+\lambda^2x_1^2)$$
$$+(x_1+2\varepsilon x_2)\log\left(\left(\gamma h+\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)\right)^2+\lambda^2x_1^2+4\varepsilon\lambda^2x_1x_2+4\varepsilon^2\lambda^2x_2^2\right)\Big]$$
$$+\frac{b_0}{2\pi D}\Big[(\gamma x_2+\varepsilon x_1)\tan^{-1}\left(\frac{\gamma h-\gamma x_2-\varepsilon x_1}{\lambda x_1}\right)$$
$$+\left(-\gamma x_2+\varepsilon(x_1+2\varepsilon x_2)\right)\tan^{-1}\left(\frac{\gamma h+\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)}{\lambda(x_1+2\varepsilon x_2)}\right)\Big]$$
$$-\frac{\lambda b_0}{4\pi\gamma^2D^2}\Big[x_1(\gamma x_2+\varepsilon x_1)\log(2\varepsilon x_1(\gamma x_2-\gamma h)+(\gamma x_2-\gamma h)^2+\varepsilon^2x_1^2+\lambda^2(x_1+2\varepsilon x_2)_1^2)$$
$$+\gamma h$$
$$+(x_1+2\varepsilon x_2)(-\gamma x_2$$
$$+\varepsilon(x_1+2\varepsilon x_2))\log\left(\left(\gamma h+\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)\right)^2+\lambda^2x_1^2+4\varepsilon\lambda x_1x_2+4\varepsilon^2\lambda^2x_2^2\right)+\gamma h$$
$$+\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)\Big]$$
$$-\frac{b_0}{4\pi\gamma^2D^2}\Big[(\gamma^2x_2^2+2\gamma\varepsilon x_1x_2+\varepsilon^2x_1^2-\lambda^2x_1^2)\tan^{-1}\left(\frac{\gamma h-\gamma x_2-\varepsilon x_1}{\lambda x_1}\right)$$
$$+(\gamma^2x_2^2-2\gamma\varepsilon(x_1+2\varepsilon x_2)x_2+\varepsilon^2(x_1+2\varepsilon x_2)^2$$
$$-\lambda^2(x_1$$
$$+2\varepsilon x_2)^2)\tan^{-1}\left(\frac{\gamma h+\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)}{\lambda(x_1+2\varepsilon x_2)}\right)\Big] \quad (26)$$

Substituting limits in (25), we obtain



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$$\omega(x) = -\frac{b_0}{4\pi\gamma^2 D^2}\Bigg[(2\gamma^2 D(D-\gamma x_2) - 2\gamma\varepsilon x_1(\gamma D - x_2) + (\varepsilon^2-\lambda^2)x_1^2$$
$$+\gamma^2 x_2^2)\tan^{-1}\frac{\lambda W x_1}{x_1^2+\varepsilon x_1(2x_2-W)-\gamma x_2(W-x_2)}\Bigg]$$
$$-\frac{b_0}{4\pi\gamma^2 D^2}\Bigg[(2\gamma^2 D(D+\gamma x_2) - 2\gamma\varepsilon(x_1+2\varepsilon x_2)(\gamma D + x_2) + (\varepsilon^2-\lambda^2)(x_1+2\varepsilon x_2)^2$$
$$+\gamma^2 x_2^2)\tan^{-1}\frac{\lambda W(x_1+2\varepsilon x_2)}{(x_1+2\varepsilon x_2)^2-\varepsilon(x_1+2\varepsilon x_2)(2x_2-W)-\gamma x_2(W+x_2)}\Bigg]$$
$$-\frac{b_0}{4\pi\gamma^2 D^2}\Bigg[\lambda x_1(\gamma(D-x_2)-\varepsilon x_1)\log\frac{(\gamma W-\gamma x_2-\varepsilon x_1)^2+\lambda^2 x_1^2}{(\gamma x_2+\varepsilon x_1)^2+\lambda^2 x_1^2}\Bigg]$$
$$-\frac{b_0}{4\pi\gamma^2 D^2}\Bigg[\lambda(x_1+2\varepsilon x_2)\big(\gamma(D+x_2)$$
$$-\varepsilon(x_1+2\varepsilon x_2)\big)\log\frac{\big(\gamma W+\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)\big)^2+\lambda^2x_1^2+4\varepsilon\lambda x_1x_2+4\varepsilon^2\lambda^2x_2^2}{\big(\gamma x_2-\varepsilon(x_1+2\varepsilon x_2)\big)^2+\lambda^2x_1^2+4\varepsilon\lambda x_1x_2+4\varepsilon^2\lambda^2x_2^2}\Bigg]$$
$$+\frac{b_0}{2\pi\gamma D^2}\lambda W(x_1+\varepsilon x_2) \tag{27}$$

Defining the dimensionless displacement and width – depth ratio through the following relations

$$U = {}^{\omega(x)}/_{b_0},\quad W_D = {}^{W}/_{D},\qquad X=\frac{x_1}{b_0},\ Y=\frac{x_2}{b_0} \tag{28}$$

Equation (27) reduces to

$$U = -\frac{1}{4\pi\gamma^2}\Bigg[(2\gamma^2 W_D(W_D-\gamma Y) - 2\gamma\varepsilon X(\gamma W_D - Y) + (\varepsilon^2-\lambda^2)X^2$$
$$+\gamma^2Y^2)\tan^{-1}\frac{\lambda W_D X}{X^2+\varepsilon X(2Y-W_D)-\gamma Y(W_D-Y)}\Bigg]$$
$$-\frac{1}{4\pi\gamma^2}\Bigg[(2\gamma^2 W_D(W_D+\gamma Y) - 2\gamma\varepsilon(X+2\varepsilon Y)(\gamma W_D + Y) + (\varepsilon^2-\lambda^2)(X+2\varepsilon Y)^2$$
$$+\gamma^2Y^2)\tan^{-1}\frac{\lambda W_D(X+2\varepsilon Y)}{(X+2\varepsilon Y)^2-\varepsilon(X+2\varepsilon Y)(2Y-W_D)-\gamma Y(W_D+Y)}\Bigg]$$
$$-\frac{1}{4\pi\gamma^2}\Big[\lambda X(\gamma(W_D-Y)-\varepsilon X)\log\frac{(\gamma W_D-\gamma Y-\varepsilon X)^2+\lambda^2X^2}{(\gamma Y+\varepsilon X)^2+\lambda^2X^2}\Big]-\frac{1}{4\pi\gamma^2}\Big[\lambda(X+2\varepsilon Y)\big(\gamma(W_D+Y)-\varepsilon(X+$$
$$2\varepsilon Y)\big)\log\frac{\big(\gamma W_D+\gamma Y-\varepsilon(X+2\varepsilon Y)\big)^2+\lambda^2X^2+4\varepsilon XY+4\varepsilon^2\lambda^2Y^2}{\big(\gamma Y-\varepsilon(X+2\varepsilon Y)\big)^2+\lambda^2X^2+4\varepsilon XY+4\varepsilon^2\lambda^2Y^2}\Big]+\frac{1}{2\pi\gamma}\lambda(X+$$
$$\varepsilon Y) \tag{29}$$

#### 2.1.1 Orthotropic Case

For an orthotropic elastic medium, we take $\varepsilon = 0$ . Equation (29) reduces to

$$U_{ortho} = -\frac{1}{4\pi\gamma}\left[(2\gamma W_D(W_D - \gamma Y) - X^2 + \gamma Y^2 + Y^2)\tan^{-1}\frac{\sqrt{\gamma}W_D X}{X^2 - \gamma Y(W_D - Y)}\right]$$
$$-\frac{1}{4\pi\gamma}\left[(2\gamma W_D(W_D + \gamma Y) - X^2 + \gamma Y^2 + Y^2)\tan^{-1}\frac{\sqrt{\gamma}W_D X}{X^2 - \gamma Y(W_D + Y)}\right]$$
$$-\frac{1}{4\pi\gamma}\left[X\sqrt{\gamma}(W_D - Y)\log\frac{\gamma(W_D - Y)^2 + X^2}{Y^2 + X^2}\right] - \frac{1}{4\pi\gamma}\left[X\sqrt{\gamma}(W_D + Y)\log\frac{\gamma(W_D + Y)^2 + X^2}{Y^2 + X^2}\right]$$
$$+\frac{1}{2\pi}X \tag{30}$$

### 2.1.2 Isotropic Case

For an isotropic elastic medium, we take $\varepsilon = 0,\ \gamma = 1$. The equation (29) reduces to

$$U_{iso} = -\frac{1}{4\pi}\left[(2W_D(W_D - Y) - X^2 + Y^2)\tan^{-1}\frac{W_D X}{X^2 - Y(W_D - Y)}\right]$$
$$-\frac{1}{4\pi}\left[(2W_D(W_D + Y) - X^2 + Y^2)\tan^{-1}\frac{W_D X}{X^2 - Y(W_D + Y)}\right]$$
$$-\frac{1}{4\pi}\left[X(W_D - Y)\log\frac{(W_D - Y)^2 + X^2}{Y^2 + X^2}\right] - \frac{1}{4\pi}\left[X(W_D + Y)\log\frac{(W_D + Y)^2 + X^2}{Y^2 + X^2}\right]$$
$$+\frac{1}{2\pi}X \tag{31}$$

### Special Case

When $D$ is sufficiently large in monoclinic medium, i.e. as $D \to \infty$ in equation (27) we obtain

$$\omega(x) = -\frac{b_0}{2\pi}\left[\tan^{-1}\left(\frac{\gamma W - \gamma x_2 - \varepsilon x_1}{\lambda x_1}\right) + \tan^{-1}\left(\frac{\gamma W + \gamma x_2 - \varepsilon(x_1 + 2\varepsilon x_2)}{\lambda(x_1 + 2\varepsilon x_2)}\right) + \tan^{-1}\left(\frac{\gamma x_2 + \varepsilon x_1}{\lambda x_1}\right) - \tan^{-1}\left(\frac{\gamma x_2 - \varepsilon(x_1 + 2\varepsilon x_2)}{\lambda(x_1 + 2\varepsilon x_2)}\right)\right] \tag{32}$$

The above result coincides with the corresponding result of uniform discontinuity after taking $\delta = 90°$ and d = 0 as a particular case in the equation (28) earlier obtained by Singh et al (2003).

### 2.2 Model II (Two Monoclinic Elastic Semi-Infinite Media )

Here, we consider a homogeneous anisotropic elastic infinite medium consisting of two monoclinic semi-infinite elastic media. The lower half space $(M_1 : x_2 > 0)$ is represented as medium *I* and the upper half space $(M_2 : x_2 < 0)$ is represented as medium *II* with $x_2$ − axis vertically downward. The interface represents the origin of the Cartesian coordinate system $Ox_1x_2x_3$. We further assume that both elastic media are homogeneous and monoclinic with $x_3 = 0$ as the symmetry plane. The interface between two half spaces may either be smooth rigid, rough rigid or perfectly bonded

#### Interface conditions

i. When the interface $x_2 = 0$ is of smooth rigid type, the condition is

$$\sigma_{32}(x_2 = 0) = 0 \tag{33}$$

ii. When the interface $x_2 = 0$ is of rough rigid type, the condition is

$$\omega(x_2 = 0) = 0 \tag{34}$$

iii. When the interface $x_2 = 0$ is perfectly bonded, the continuity of displacement and shear stresses implies

$$\omega(x_2 < 0) = \omega(x_2 > 0),$$
$$\sigma_{32}(x_2 < 0) = \sigma_{32}(x_2 > 0) \qquad (35)$$

Let there be an exponential variable slip along the fault with fault depth $D$, infinite length $-\infty < x_3 < \infty$ and finite width $W$, situated on $x_2$ −axis with which is taken as vertically downward as shown in figure 2.

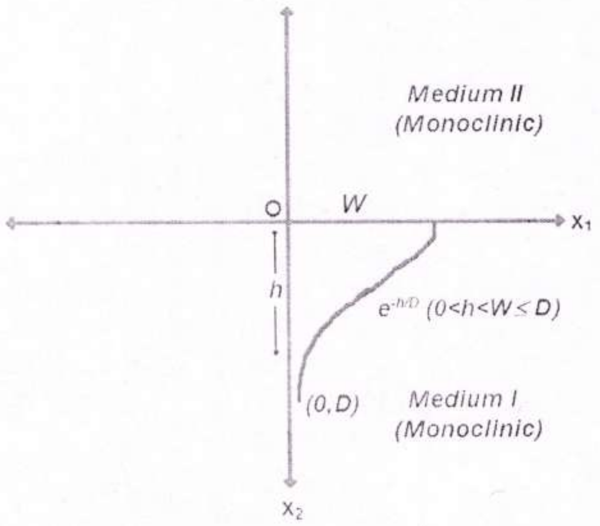


Figure 2 Geometry of the Problem

Following Ting (1995), the elastic fields as a result of F (line force) acting at $(0, d)$ of a monoclinic semi-infinite medium in welded contact with another monoclinic semi-infinite medium are given by

$$\omega^{(1)} = -\frac{F}{2\pi M^{(1)}} \mathrm{R}\left\{\ln\left(Z^{(1)} - q^{(1)}d\right) - \ln\left(Z^{(1)} - \bar{q}^{(1)}d\right)\right\} \qquad (36)$$

$$\omega^{(2)} = -\frac{F(1+K)}{2\pi M^{(2)}} \mathrm{R}\ \ln\left(Z^{(2)} - q^{(1)}d\right) \qquad (37)$$

where R denotes the real part and an over bar over $q^{(1)}$ denotes complex conjugate of $q^{(1)}$ and the superscript (1) is for Medium *I* and superscript (2) is for Medium *II*. Here,

$$K = \frac{M^{(2)} - M^{(1)}}{M^{(2)} + M^{(1)}}, \quad (-1 < K < 1) \qquad (38)$$

$$Z^{(n)} = x_1 + q^{(n)}x_2, \quad n = (1, 2) \qquad (39)$$

$$q^{(n)} = \frac{-c_{45}^{(n)} + iM^{(n)}}{c_{44}^{(n)}}, \quad i = \sqrt{-1} \qquad (40)$$



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$$M^{(n)} = \left(C_{44}^{(n)}C_{55}^{(n)} - C_{45}^{(n)^2}\right)^{1/2} > 0 \quad (41)$$

For the Medium *I*, $\omega^{(1)}$ is the horizontal displacement and $C_{mn}^{(1)}$ are elastic stiffnesses and for Medium *II*, $\omega^{(2)}$ is the horizontal displacement and $C_{mn}^{(2)}$ are elastic stiffnesses. Equations (36) and (37)satisfies the boundary conditions

$$\omega^{(1)} = \omega^{(2)} \quad \text{at} \quad x_2 = 0 \quad (42)$$

For the perfect contact, the displacements at any point are given by (Kumar, 2015)

$$\omega^{(1)} = \frac{\lambda_1}{2\pi}\int_0^L \left(\frac{Y_5}{R_1^2} + K\frac{Y_6}{S_1^2}\right) b ds \quad (43)$$

$$\omega^{(2)} = \frac{1-K}{2\pi}\int_0^L \frac{Y_7}{R_2^2} b ds \quad (44)$$

where

$$R_1^2 = (A + \varepsilon_1 \sin 2\delta)s^2 - 2[Cx + B(x_2 - d)]s + x_1^2 + \gamma_1(x_2 + d)^2 + 2\varepsilon_1 x_1(x_2 - d) \quad (45)$$

$$S_1^2 = (A + \varepsilon_1 \sin 2\delta)s^2 - 2\left[C\left(x_1 + 2\varepsilon_1 x_2 - B(x_2 + d)\right)\right]s + x_1^2 + \gamma_1(x_2 - d)^2 + 2\varepsilon_1 x_1(x_2 - d) - 4\varepsilon_1^2 x_2 d \quad (46)$$

$$R_2^2 = (A + \varepsilon_1 \sin 2\delta)s^2 - 2[Cx_1 + (C\varepsilon_2 + \lambda_1\lambda_2 \sin\delta)x_2 - Bd]s + x_1^2 + \gamma_2 x_2^2 + \gamma_1 d^2 + 2\varepsilon_2 x_1 x_2 - 2\varepsilon_1 x_1 d - 2(\lambda_1\lambda_2 + \varepsilon_1\varepsilon_2)dx_2 \quad (47)$$

$$Y_5 = -x_1 \sin\delta + (x_2 - d)\cos\delta \quad (48)$$

$$Y_6 = (x_1 + 2\varepsilon_1 x_2)\sin\delta + (x_2 + d)\cos\delta \quad (49)$$

$$Y_7 = -\lambda_1 x_1 \sin\delta + x_2(\lambda_2 \cos\delta - (\lambda_1\varepsilon_2 - \lambda_2\varepsilon_1)\sin\delta) - \lambda_1 \cos\delta d \quad (50)$$

$$A_1 = \cos^2\delta + \gamma_1 \sin^2\delta \quad (51)$$

$$B_1 = \varepsilon_1 \cos\delta + \gamma_1 \sin\delta \quad (52)$$

$$C_1 = \cos\delta + \varepsilon_1 \sin\delta \quad (53)$$

In figure 2, exponential variable slip along the fault has the same behavior as that in fig.1 and using equation (4), equations (45 – 53) yield

$$R_1^2 = \frac{1}{A}\{[Ah - Cx_1 - Bx_2]^2 + \lambda^2 x_1^2\} \quad (54)$$

$$S_1^2 = \frac{1}{A}\{[Ah - C(x_1 + 2\varepsilon x_2) + Bx_2]^2 + \lambda^2(x_1 + 2\varepsilon x_2)^2\} \quad (55)$$

$$R_2^2 = \frac{1}{A}\{-[Ah - Cx_1 - (C\varepsilon_2 + \lambda_1\lambda_2)x_2]^2 + [\lambda_1 x_1 + x_2(\lambda_1\varepsilon_2 - \lambda_2\varepsilon_1)]^2\} \quad (56)$$

$$Y_5 = -x_1 \quad (57)$$

$$Y_6 = (x_1 + 2\varepsilon_1 x_2) \quad (58)$$

$$Y_7 = -\lambda_1 x_1 - x_2(\lambda_1\varepsilon_2 - \lambda_2\varepsilon_1) \quad (59)$$

$$A_1 = B_1 = \gamma_1, \quad C_1 = \varepsilon_1 \quad (60)$$

Substituting equations (55 – 61) in equation (43) and equation (44), we get

$$\omega^{(1)} = \frac{1}{2\pi M^{(1)}} \int_L \exp\left(-{}^{h}/_{D}\right) \Big| -C_{55}^{(1)}\{x_1 + x_1(x_2 - h)\}\left(\frac{1}{R_1^2} - \frac{K}{S_1^2}\right)$$
$$- C_{45}^{(1)}\left\{\varepsilon_1\left(x_1 + \varepsilon_1(x_2 - h)\right)\left(\frac{1}{R_1^2} - \frac{K}{S_1^2}\right)\right.$$
$$\left. + \lambda_1^2\left(\frac{x_2 - h}{R_1^2} + K\frac{x_2 + h}{S_1^2}\right)\right\}\Big| ds \tag{61}$$

$$\omega^{(2)} = \frac{1 + K}{2\pi M^{(2)}} \int_L \exp\left(-{}^{h}/_{D}\right) \left[-C_{55}^{(1)}(x_1 + \varepsilon_2 x_2 - \varepsilon_1 h)\right.$$
$$\left. - C_{45}^{(1)}\ \{\varepsilon_1(\varepsilon_2 x_2 - \varepsilon_1 h) + \lambda_1(\lambda_2 x_2 - \lambda_1 h)\}\right] \frac{1}{R_2^2} ds \tag{62}$$

where

$$\varepsilon_1 = -C_{45}^{(1)}/C_{44}^{(1)}, \quad \varepsilon_2 = -C_{45}^{(2)}/C_{44}^{(2)} \tag{63}$$

$$\gamma_1 = C_{55}^{(1)}/C_{44}^{(1)}, \quad \gamma_2 = C_{55}^{(2)}/C_{44}^{(2)} \tag{64}$$

$$\lambda_1 = (\gamma_1 - \varepsilon_1^2)^{1/2}, \quad \lambda_2 = (\gamma_2 - \varepsilon_2^2)^{1/2} \tag{65}$$

Expanding $\exp\left(-{}^{h}/_{D}\right)$ by Taylor Infinite Series Expansion as in equation (23) and neglecting the higher power terms of $\frac{h}{D}$ $(\geq 3)$ and substituting in equation (61) and equation (62) gives

$$\omega^{(1)} = \frac{1}{2\pi M^{(1)}} \int_L b_0\left(1 - \frac{h}{D} + \frac{h^2}{2D^2}\right) \Big| -C_{55}^{(1)}\{x_1 + x_1(x_2 - h)\}\left(\frac{1}{R_1^2} - \frac{K}{S_1^2}\right)$$
$$- C_{45}^{(1)}\left\{\varepsilon_1\left(x_1 + \varepsilon_1(x_2 - h)\right)\left(\frac{1}{R_1^2} - \frac{K}{S_1^2}\right)\right.$$
$$\left. + \lambda_1^2\left(\frac{x_2 - h}{R_1^2} + K\frac{x_2 + h}{S_1^2}\right)\right\}\Big| ds \tag{66}$$

$$\omega^{(2)} = \frac{1 + K}{2\pi M^{(2)}} \int_L b_0\left(1 - \frac{h}{D} + \frac{h^2}{2D^2}\right) \left[-C_{55}^{(1)}(x_1 + \varepsilon_2 x_2 - \varepsilon_1 h)\right.$$
$$\left. - C_{45}^{(1)}\ \{\varepsilon_1(\varepsilon_2 x_2 - \varepsilon_1 h) + \lambda_1(\lambda_2 x_2 - \lambda_1 h)\}\right] \frac{1}{R_2^2} ds \tag{67}$$

Integrating equations (66) and (67) and using the dimensionless relations

$$\omega^{(1)} = b_0 U^{(1)}, \qquad \omega^{(2)} = b_0 U^{(2)} \tag{68}$$

we obtain the explicit expressions for displacement as

$$U^{(1)} = -\frac{1}{4\pi\gamma_1^2}\Bigg[(2\gamma_1^2 W_D(W_D - \gamma_1 Y) - 2\gamma_1\varepsilon_1 X(\gamma_1 W_D - Y) + \gamma_1^2 Y^2 + \varepsilon_1^2 X^2$$

$$- \lambda_1^2 X^2)\tan^{-1}\frac{\gamma_1 W_D \lambda_1 X}{(\lambda_1^2 + \varepsilon_1^2)X^2 - \gamma_1^2 Y(W_D - Y) + (\gamma_1\varepsilon_1 X(2Y - W_D))}\Bigg]$$

$$+ \frac{K}{4\pi\gamma_1^2}\Bigg[(-2\gamma_1^2 W_D(W_D + \gamma_1 Y) + 2\gamma_1\varepsilon_1(X + 2\varepsilon_1 Y)(\gamma_1 W_D - Y) + \gamma_1^2 Y^2 + \varepsilon_1^2(X + 2\varepsilon_1 Y)^2$$

$$- \lambda_1^2(X$$

$$+ 2\varepsilon_1 Y)^2)\tan^{-1}\frac{\gamma_1 W_D \lambda_1(X + 2e_1 Y)}{(\lambda_1^2 + \varepsilon_1^2)(X + 2\varepsilon_1 Y)^2 + \gamma_1^2 Y(W_D + Y) - (\gamma_1\varepsilon_1(X + 2\varepsilon_1 Y)(2Y - W_D))}\Bigg]$$

$$- \frac{\lambda_1 X}{4\pi\gamma_1^2}\Bigg[(\gamma_1(W_D - Y) - \varepsilon_1 X)\log\frac{(\gamma_1 W_D - \gamma_1 Y - \varepsilon_1 X)^2 + \lambda_1^2 X^2}{(\gamma_1 Y + \varepsilon_1 X)^2 + \lambda_1^2 X^2}\Bigg]$$

$$+ \frac{K\lambda_1(X + 2\varepsilon_1 Y)}{4\pi\gamma_1^2}\Bigg[(\gamma_1(W_D - Y)$$

$$+ \varepsilon_1(X + 2\varepsilon_1 Y))\log\frac{(\gamma_1 W_D + \gamma_1 Y - \varepsilon_1(X + 2\varepsilon_1 Y))^2 + \lambda_1^2 X^2 + 4\varepsilon_1\lambda_1 XY + 4\varepsilon_1^2\lambda_1^2 Y^2}{(\gamma_1 Y - \varepsilon_1(X + 2\varepsilon_1 Y))^2 + \lambda_1^2 X^2 + 4\varepsilon_1\lambda_1 XY + 4\varepsilon_1^2\lambda_1^2 Y^2}\Bigg]$$

$$+ \frac{\lambda_1}{4\pi\gamma_1}[X + K(X + 2\varepsilon_1 Y)] \qquad (69)$$

$$U^{(2)} = \frac{(1-K)}{4\pi\gamma_1^2}\Big[\big(2\gamma_1^2 {W_D}^2 - 2\gamma_1 W_D(\varepsilon_1(\varepsilon_2 Y + X) + \lambda_1\lambda_2 Y) + \varepsilon_1^2(\varepsilon_2 Y + X)^2 + 4\varepsilon_1\lambda_1\lambda_2 Y(\varepsilon_2 Y + X) +$$

$$\lambda_1^2(-(Y^2(\varepsilon_2^2 - \lambda_2^2) + 2\varepsilon_2 XY + X^2)) - \varepsilon_1^2\lambda_2^2 Y^2\big)\left(\tan^{-1}\frac{\gamma_1 W_D - \varepsilon_1(\varepsilon_2 Y + X) - \lambda_1\lambda_2 Y}{\varepsilon_1\lambda_2 Y - \lambda_1(\varepsilon_2 Y + X)} + \tan^{-1}\frac{\varepsilon_1(\varepsilon_2 Y + X) + \lambda_1\lambda_2 Y}{\varepsilon_1\lambda_2 Y - \lambda_1(\varepsilon_2 Y + X)}\right)\Big] +$$

$$\frac{(1-K)}{4\pi\gamma_1^2}\Big[(Y(\lambda_2\varepsilon_1 - \lambda_1\varepsilon_2) - \lambda_1 X)\big(\varepsilon_1(\varepsilon_2 Y + X) + \lambda_1\lambda_2 W_D Y -$$

$$\gamma_1 {W_D}^2\big)\log\frac{(\gamma_1 W_D - \varepsilon_1(\varepsilon_2 Y + X) - \lambda_1\lambda_2 Y)^2 + \varepsilon_2^2\lambda_1^2 Y^2 + 2\varepsilon_2\lambda_1^2 XY - 2\varepsilon_1\lambda_1\lambda_2 XY - 2\varepsilon_1\varepsilon_2\lambda_1\lambda_2 Y^2 + \lambda_1^2 X^2 + \varepsilon_1^2\lambda_1^2 Y^2}{(\varepsilon_1(\varepsilon_2 Y + X) - \lambda_1\lambda_2 Y)^2 + \varepsilon_2^2\lambda_1^2 Y^2 + 2\varepsilon_2\lambda_1^2 XY - 2\varepsilon_1\lambda_1\lambda_2 XY - 2\varepsilon_1\varepsilon_2\lambda_1\lambda_2 Y^2 + \lambda_1^2 X^2 + \varepsilon_1^2\lambda_1^2 Y^2}\Big]$$

$$(70)$$

### 2.2.1 Orthotropic Elastic Medium

When the medium I is orthotropic, we take $\varepsilon_1 = 0$ in equation (70) and obtain the corresponding displacement:

$$U^{(1)}_{ortho} = -\frac{1}{4\pi\gamma_1}\left[(2\gamma_1 W_D(W_D - \gamma_1 Y) + \gamma_1 Y^2 - X^2)\tan^{-1}\frac{\sqrt{\gamma_1}W_D X}{X^2 - \gamma_1 Y(W_D - Y)}\right]$$

$$+ \frac{K}{4\pi\gamma_1}\left[(-2\gamma_1 W_D(W_D + \gamma_1 Y) + \gamma_1 Y^2 - X^2)\tan^{-1}\frac{\sqrt{\gamma_1}W_D X}{X^2 - \gamma_1 Y(W_D + Y)}\right]$$

$$- \frac{1}{4\pi\gamma_1}\left[\sqrt{\gamma_1}X(W_D - Y)\log\frac{\gamma_1(W_D - Y)^2 + X^2}{\gamma_1 Y^2 + X^2}\right]$$

$$+ \frac{K}{4\pi\gamma_1}\left[\sqrt{\gamma_1}X(W_D + Y)\log\frac{\gamma_1(W_D + Y)^2 + X^2}{\gamma_1 Y^2 + X^2}\right]$$

$$+ \frac{1}{4\pi\gamma_1}[\sqrt{\gamma_1}X(1 + K)] \qquad (71)$$

### 2.2.2 Isotropic Elastic Medium

When the medium I is isotropic, we take $\varepsilon_1 = 0,\ \gamma_1 = 1$ in equation (70) and obtain the corresponding displacement as:

$$
\begin{aligned}
U_{iso}^{(1)} = &-\frac{1}{4\pi}\left[(2W_D(W_D - Y) + Y^2 - X^2)\tan^{-1}\frac{W_D X}{Y^2 - Y(W_D - Y)}\right] \\
&+\frac{K}{4\pi}\left[(-2W_D(W_D + Y) + Y^2 - X^2)\tan^{-1}\frac{W_D X}{X^2 - Y(W_D + Y)}\right] \\
&-\frac{1}{4\pi}\left[X(W_D - Y)\log\frac{(W_D - Y)^2 + X^2}{Y^2 + X^2}\right] + \frac{K}{4\pi}\left[X(W_D + Y)\log\frac{(W_D + Y)^2 + X^2}{Y^2 + X^2}\right] \\
&+\frac{1}{4\pi}[X(1 + K)]
\end{aligned} \tag{72}
$$

Special Case

Taking $D$ as sufficiently large (i.e. as $D \to \infty$), we obtained

$$
\begin{aligned}
&\omega^{(1)} \\
&= \frac{b_0}{2\pi}\left[\tan^{-1}\frac{-\gamma_1 h + \gamma_1 x_2 + \varepsilon_1 x_1}{\lambda_1 x_1} - \tan^{-1}\frac{\gamma_1 x_2 + \varepsilon_1 x_1}{\lambda_1 x_1}\right] \\
&+ \frac{b_0 K}{2\pi}\left[\tan^{-1}\frac{\gamma_1 h + \gamma_1 x_2 - \varepsilon_1(x_1 + 2\varepsilon_1 x_2)}{\lambda_1(x_1 + 2\varepsilon_1 x_2)}\right. \\
&\left. - \tan^{-1}\frac{\gamma_1 x_2 - \varepsilon_1(x_1 + 2\varepsilon_1 x_2)}{\lambda_1(x_1 + 2\varepsilon_1 x_2)}\right]
\end{aligned} \tag{73}
$$

$$
\omega^{(2)} = \frac{(1 - K)b_0}{2\pi}\tan^{-1}\frac{\gamma_1 h - \varepsilon_1(\varepsilon_2 x_2 + x_1) - \lambda_1\lambda_2 x_2}{\varepsilon_1\lambda_2 x_2 - \lambda_1(x_1 + \varepsilon_2 x_2)} \tag{74}
$$

The above results coincide with the corresponding results of uniform discontinuity after taking $\delta = 90°$ and $d = 0$ as a particular case in the equation (33) and (34) earlier obtained by Kumar et al (2002).

## 3. Numerical Results and Discussions

Here, we use the different values of anisotropic parameters and obtain the change in the displacement with the change in the horizontal distance from the fault trace at surface and sub-surface levels by using MATLAB graphical routines and to investigate the effect due to W-D ratio, we take the different values of width-depth ratios as $W' = W_D = 0.3, 0.5$ and $0.8$.

For numerical purpose, let the material for monoclinic elastic half-space be Dolomite (Madan *et al*, 2011). The change in displacement with change in horizontal distance from the fault trace for different values of $W_D$ are depicted in figures $(3 - 5)$. These figures are depicting the change in displacement $U$ at $Y = 0$ i.e.(surface), $Y = 0.5$ i.e.(subsurface) and at the depth point $Y = 1$ for Dolomite.

At $Y = 0$, the discontinuity is same for all the three values of $W_D$ ratio and is equal to 12 in magnitude. At $Y = 0.5$, the discontinuity decreases to 8 in magnitude for $W_D = 0.5, 0.8$ whereas displacement becomes continuous for $W_D = 0.3$. At $Y = 1$, displacement is continuous for all values of $W_D$. It is interesting to note that the effect of discontinuity disappears as we move away from the fault.

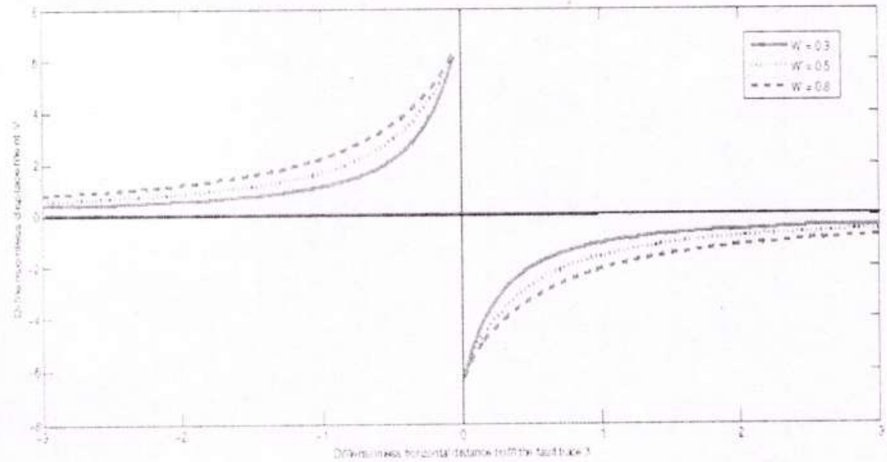


Fig. 3 Monoclinic Semi-infinite Medium: $Y = 0,\ \gamma_1 = 1.043,\ \varepsilon_1 = -0.063.$
(Dolomite)

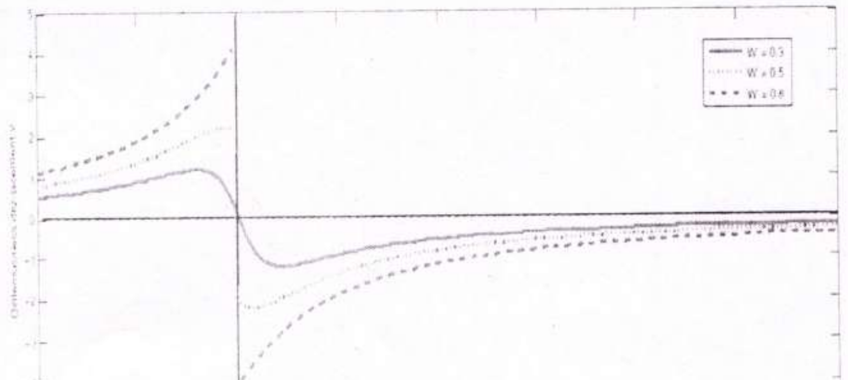


Fig. 4 Monoclinic Semi-infinite Medium : $Y = 0.5,\ \gamma_1 = 1.043, \varepsilon_1 = -0.063.$
( Dolomite)



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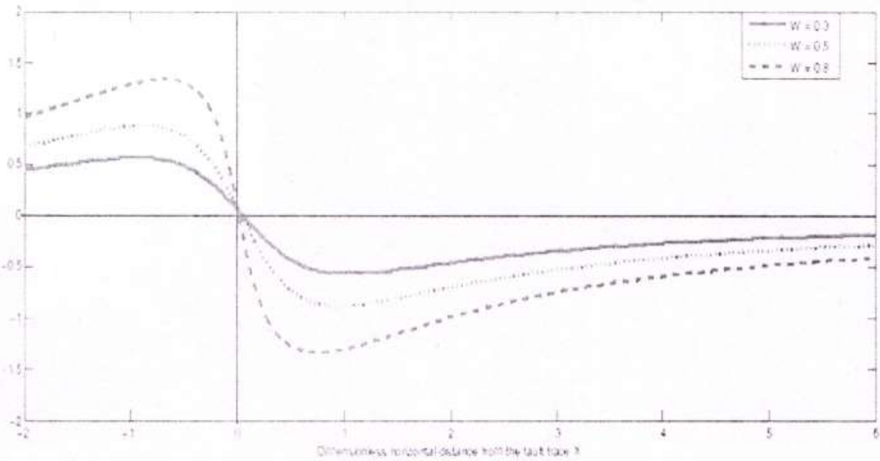

Fig. 5 Monoclinic Semi-infinite Medium : $Y = 1$, $\gamma_1 = 1.043$, $\varepsilon_1 = -0.063$. (Dolomite)

### 3.1 Variation of Fault with Width-Depth Ratio

By making use of the Dolomite material given by Madan *et al* (2011), Olivine material given by Verma (1960) and Topaz given by Love (1944), the graphs are plotted in MATLAB graphical routine. To examine the effect of the fault- variation with width-depth ratio in case of two perfectly bonded anisotropic elastic half-spaces, the medium *I* is considered as Dolomite which is monoclinic and medium *II* is considered as Topaz i.e. Orthotropic Elastic medium. For different values of $W_D$, the variations of the displacement with horizontal distance are shown in figures $(6 - 11)$. Fig. 6 shows that the discontinuity is equal to 2 in magnitude for all $W_D$ ratios ($W_D = 0.3, 0.5$ and $0.8$) at the surface $Y = 0$ in Medium *I* of monoclinic two-half spaces (Medium *I*: Dolomite, Medium *II*: Topaz). Further, at the subsurface $Y = 0.5$ (Fig. 7), the discontinuity increases to 8 in magnitude for different $W_D$ ratios and significant difference increases for $W_D = 0.5$ and $0.8$ whereas it becomes continuous for $W_D = 0.5$ and at $Y = 1$ (Fig. 8), the displacements are continuous for all $W_D$ ratios.

Fig. 9 shows that the discontinuity is equal to 2.5 in magnitude and increases rapidly for all $W_D$ ratios ($W_D = 0.3, 0.5$ and $0.8$) at the surface ($Y = 0$) in Medium *II* of monoclinic two-half spaces (Medium *I*: Dolomite, Medium *II*: Topaz). Further, at $Y = 0.5$ (Fig. 10), the discontinuity is 1.5 in magnitude for different $W_D$ ratios and significant difference decreases rapidly and at $Y = 1$ (Fig. 11), the discontinuity decreases rapidly.

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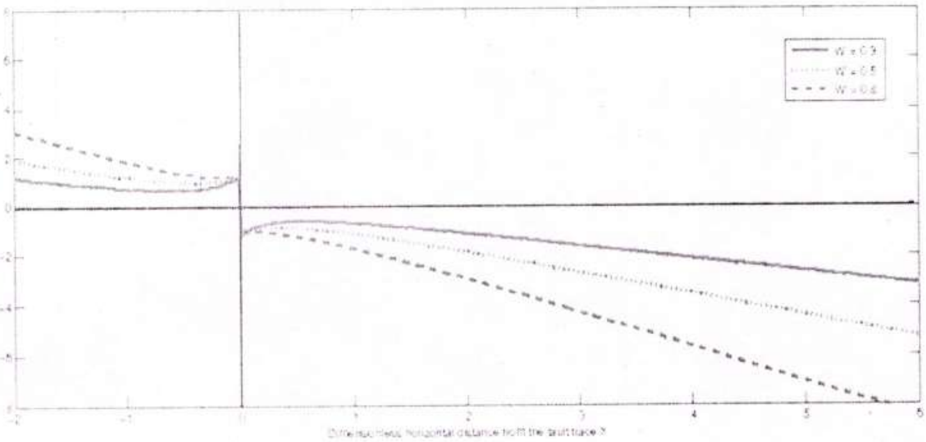

Fig. 6 Monoclinic Two-Half Spaces for $Y = 0$, $\gamma_1 = 1.043$, $\varepsilon_1 = -0.063$, K = 0.644.
(Topaz-Dolomite)

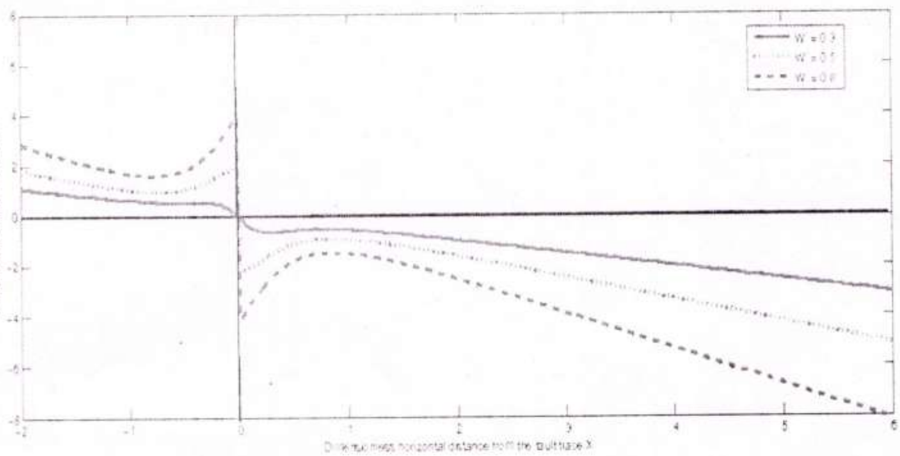

Fig. 7 Monoclinic Two-Half Spaces: $Y = 0.5$, $\gamma_1 = 1.043$, $\varepsilon_1 = -0.063$, K = 0.644.
(Topaz-Dolomite)

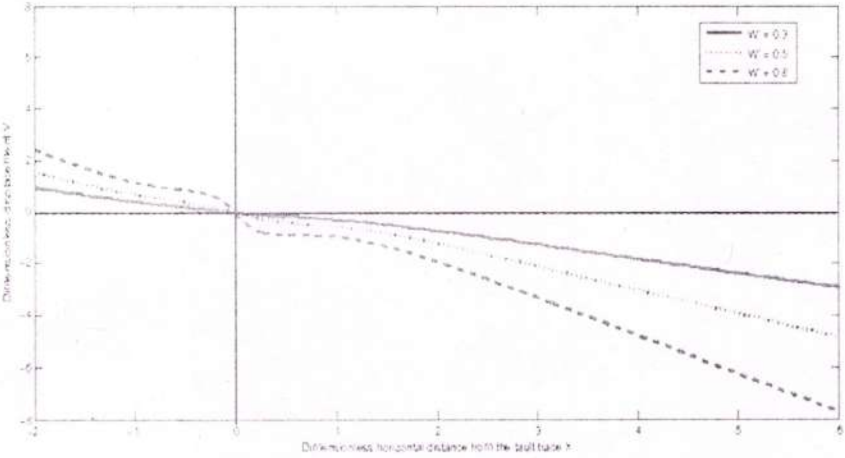

Fig. 8 Monoclinic Two-Half Spaces : $Y = 1$, $\gamma_1 = 1.043$, $\varepsilon_1 = -0.063$, K = 0.644. (Topaz-Dolomite)

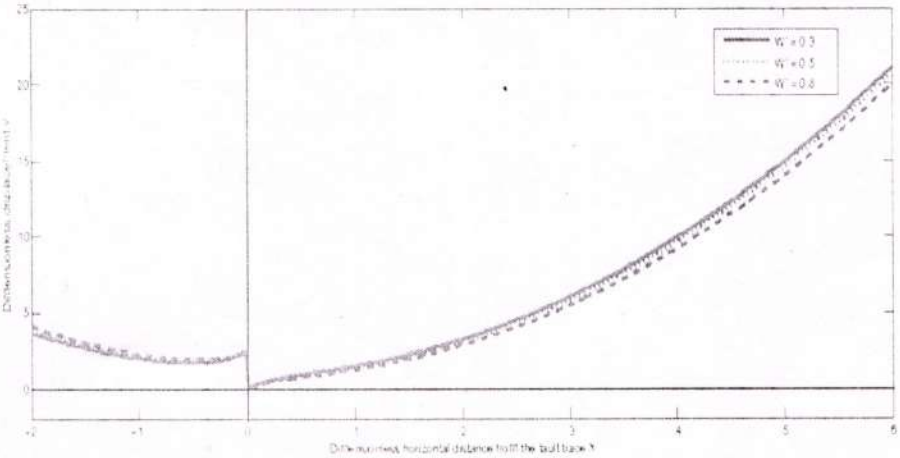

Fig. 9 Monoclinic Two-Half Spaces: $Y = 0$, $\gamma_1 = 1.043$, $\varepsilon_1 = -0.063$, $\varepsilon_2 = 0$, K = 0.644(Topaz-Dolomite)

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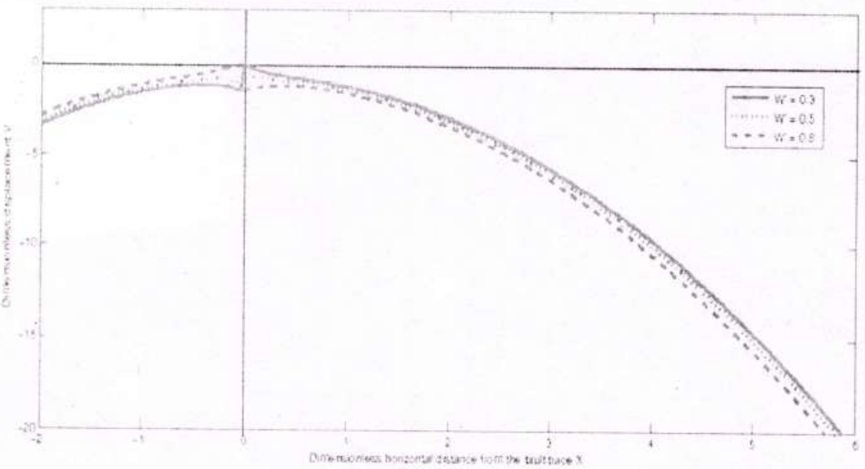

Fig. 10 Monoclinic Two-Half Spaces: $Y = 0.5$, $\gamma_1 = 1.043$, $\varepsilon_1 = -0.063$, $\varepsilon_2 = 0$, K = 0.644(Topaz-Dolomite)

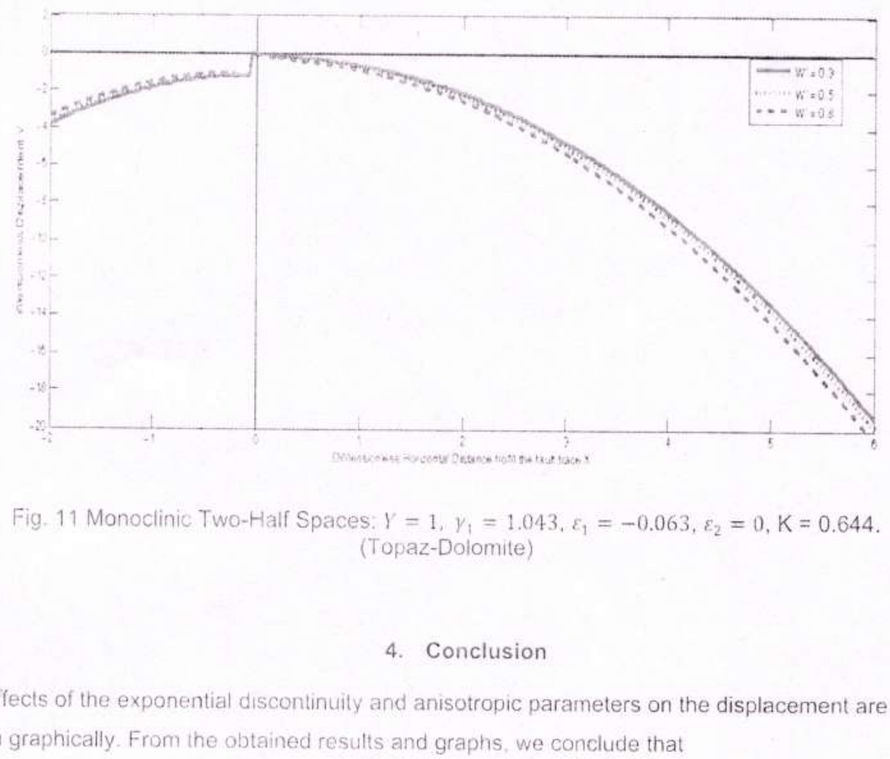

Fig. 11 Monoclinic Two-Half Spaces: $Y = 1$, $\gamma_1 = 1.043$, $\varepsilon_1 = -0.063$, $\varepsilon_2 = 0$, K = 0.644. (Topaz-Dolomite)

## 4. Conclusion

The effects of the exponential discontinuity and anisotropic parameters on the displacement are shown graphically. From the obtained results and graphs, we conclude that

1. The discontinuity decreases in magnitude for different width-depth ratios as we move from surface to some depth point and after a specific depth, the displacement becomes continuous.
2. The displacement field is significantly influenced by the nature of interface between two anisotropic elastic half-spaces. The discontinuity increases in magnitude for different values

of width-depth ratio with increasing depth and after certain depth, the displacements are continuous for all the values of width-depth ratios.

The solutions obtained for these models may be helpful in modeling the lithosphere deformation due to faulting.

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**Authored By**

Dr. Suman Nain & Ms. Seema

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# Spatio - Temporal Change in Literacy and Sex Ratio in Haryana (2001 - 2011)

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**ABSTRACT**

*Literacy and sex ratio are two important features of demography. The social and econoic development partially depends upon the level of literacy. It is assumed that literacy plays an important role for the betterment of sex ratio. Sex ratio is represented as females per thousend males. According to census of India; Average sex ratio in India was 933 in 2001 and showed a positive change in 2011 and sex ratio reached upto 940 females per thousand males. Haryana has a low sex ratio, trends show that sex ratio was 861 in 2001 and tends upwards to 879 in 2011. But still it is below the national average. Literacy being antother important aspect of demography, deals with the ability to read and write any language. This paper focuses on the correlation of literacy and sex ratio on tahsil level in Haryana. During 2001 there was a low and positive correlation between both but it became negative moderate correlation in 2011. Which shows that in Haryana both are inversely correlated.*

## 1. Introduction

In the present study the two important aspects are literacy and sex ratio an important indicator of human resource development; literacy has a number of definitions and they vary from place to place. One of the general definition is as ' the ability to read and write in any language. But one should exclude the population from 0 - 7 years while talking about literacy. Literacy is accepted as a tool that can bring social reforms, economic transformation, occupational competence and development of specific skills. Accoring to Majid Hussain (2008), " Literacy reflects the socio economic and cultural setup of a nation, ethnic group and social community.

According to census of India 2011; literacy may be defined as the ability to read and write with understanding. The other component of the study is sex ratio which represents the number of females per thousand males for the developing countries like India. While the situation is inverse for the developed countries like UK and U.S.A.. In these developed conntries sex ratio is represented as the number of males per thousand females. As both of these attributes ( sex ratio and literacy) can reflect the actual situation of the society and play an important role for various planning types for population. Both of these are correlation to each other either in the way or in negative séance. Literacy rate in 2001 and 2011 for India are 64.8 % and 74.04 % respectively while in Haryana literacy rates are comparatively high for 2001 and 2011 as 67.91% and 75.55% respectively. Sex ratio for India during census 2001 and 2011 has 933 and 940 females per 1000 males respectively.

## 2. Objectives

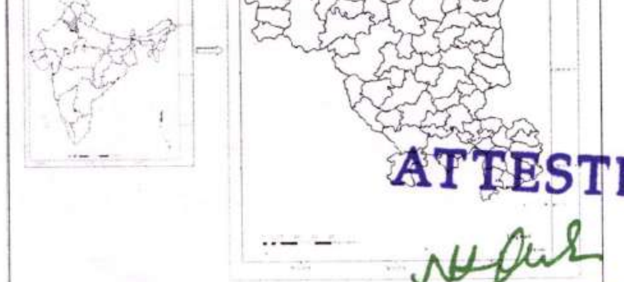
The main objectives of the study as -

- To evaluate the impact literacy on sex ratio from 2001 to 2011.
- To view the spatial change and pattern of literacy rate and sex ratio in Haryana ( tahsil wise)
- To analyse the changing correlation between literacy rate and sex ratio from 2001 to 2011.

## 3. Study Area

In terms of latitude and longitudes Haryana extends from 27°31' N to 30° 35' N latitude and 74° 28' E to 77° 36' E longitudes. It is located in the northern part of India which lies in Ganga plain. Initially it was a part of East punjab and it was carved as a new state on 1 nov.1966. Chandigarh is it's capital. The word Haryana is derived from sanskrit which means Hari ( the God Vishnu) and ayana ( Home) i.e. home of the God vishnu. Haryana is borded by Punjab, Himachal pradesh in the north direction, while the in the west and south direction Rajasthan is situated. The eastern border is well defined by river yamuna and uttar pradesh. It surrounds NCR from three sides i.e. northern, western and southern sides. Present study of Haryana deals with tahsil wise sex ratio and literacy rate from 2001 to 2011 as well as their correlation for both decades.

**Map - 1**

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### 4. Data source and Methodology

In our present study; mainly secondary data regarding sex ratio and literacy is taken from census of India for 2001 and 2011. For the analysis of spatio - temporal change in literacy rate and sex ratio, mainly spearman's Rank correlation is used. In Spearman's Rank correlation the formala is represented as

$$r = 1 - \frac{6\sum D^2}{n(n^2-1)}$$

Where D = Difference of R1 and R2

n = Total number of items

In the Spearman's Rank correlation the highest entry of both the variables are ranked as first, similarly the other enteries are also ranked. These ranks are named as R1 and R2. after calculating the ranks; difference in R1 and R2 is calculated with positive or negative symbols. Difference square is calculated for each entry point and the summation is there. Then by using the above given formula rank is calculated either positive or negative. Positive rank shows that both the indicators are correlated positively, which means that the change in one indicator will positively effect the 2nd one. and if one is increasing then the other is also increasing. The negative ranks shows that if one variable in increasing then it adversly effect the second one.

Besides this different cartographic techniques are also used for 74 tahsil of Haryana; like bar digrams choropleth map etc. The districts and tahsils of Haryana are given below in table 1.1.

Table 1.1:

| Sr. No. | Tahsil | District | Sr. No. | Tahsil | District |
|---|---|---|---|---|---|
| 1 | Naraingarh | | 38 | Narnaund | |
| 2 | Ambala | Ambala | 39 | Hansi | |
| 3 | Barara | | 40 | Bawani khera | |
| 4 | Jagadhri | | 41 | Bhiwani | |
| 5 | Bilasbur | Yamunanagar | 42 | Tosham | |
| 6 | Chhachhrauli | | 43 | Siwani | |
| 7 | Shahbad | | 44 | Loharu | Bhiwani |
| 8 | Pehowa | Kurukshetra | 45 | Dadri | |
| 9 | Thanesar | | 46 | Badhra | |
| 10 | Guhla | | 47 | Maham | |
| 11 | Kaithal | Kaithal | 48 | Rohtak | Rohtak |
| 12 | Fatehpur pundri | | 49 | Sampla | |
| 13 | Nilokheri | | 50 | Beri | |
| 14 | Indri | | 51 | Bahadurgarh | |
| 15 | Karnal | Karnal | 52 | Jhajjar | Jhajjar |
| 16 | Assadh | | 53 | Matenhail | |
| 17 | Gharaunda | | 54 | Mahendargarh | Mahendargarh |
| 18 | Panipat | | 55 | Narnaul | |
| 19 | Israna | Panipat | 56 | Kosli | |
| 20 | Samalkha | | 57 | Rewari | Rewari |
| 21 | Gohana | | 58 | Bawal | |
| 22 | Ganaur | | 59 | Pataudi | |
| 23 | Sonipat | Sonipat | 60 | Grugram | |
| 24 | Kharkhoda | | 61 | Farrukhnagar | Grugram |
| 25 | Narwana | | 62 | Manesar | |
| 26 | Jind | | 63 | Sohna | |
| 27 | Julana | Jind | 64 | Taoru | |
| 28 | Safidon | | 65 | Nuh | Mewat |
| 29 | Ratia | | 66 | Ferozepur jhirka | |

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| | | | | | |
|---|---|---|---|---|---|
| 30 | Tohana | Fatehabad | 67 | Punahana | |
| 31 | Fatehabad | | 68 | Faridabad | Faridabad |
| 32 | Dabwali | Sirsa | 69 | Ballabgarh | |
| 33 | Sirsa | | 70 | Palwal | Palwal |
| 34 | Rania | | 71 | Hathin | |
| 35 | Ellenabad | | 72 | Hodal | |
| 36 | Adampur | Hisar | 73 | Kalka | Panchkula |
| 37 | Hisar | | 74 | Panchkula | |

## 5. Results and Discussion

Literacy and sex ratio are two important indicators of human development index. Literacy is explained as ' If a person can read or write any language with understanding then that person is termed as a literate person; Here literacy rate is calculated to understand the changing scenario in Haryana from 2001 to 2011 (tahsil wise). Literacy rate is of two type as crude literacy rate and effective literacy rate. both of these can be expressed in the form of following formulas.

$$\text{Crude literacy rate} = \frac{\text{Number of literate person}}{\text{Total population}} \times 100$$

$$\text{Effective literacy rate} = \frac{\text{No. of literate persons seven and above}}{\text{Population aged seven and above}} \times 100$$

Literacy rate is explained in percentage. Crude literacy rate include whole literate population while effective literacy rate excludes the population below seven years so effective literacy rate is more useful for understanding spatio - temporal changes. Literacy rate in India is 74.04 % for census 2011, while it was 64.84% during census 2001. In case of Haryana literacy rate during 2011 and 2001 was 75.55% and 67.91% respectively. The literacy rate of Haryana (tahsil wise) for 2001 and 2011 is shown with the help of bar diagram in fig.1.

Sex composition of a population refers to the balance between male and female in any population. It can be expressed either in the form of proportion of a particular sex in the population or as a ratio between the population of two sexes. There are explaining sex ratio, the first one gives the number of males per hundred females or number of males per thousand females in the population and is the most widely used measure of sex ratio the world over. On the country the second provides the number of females per hundred males or number of females per thousand males in the population. In India second method is used:

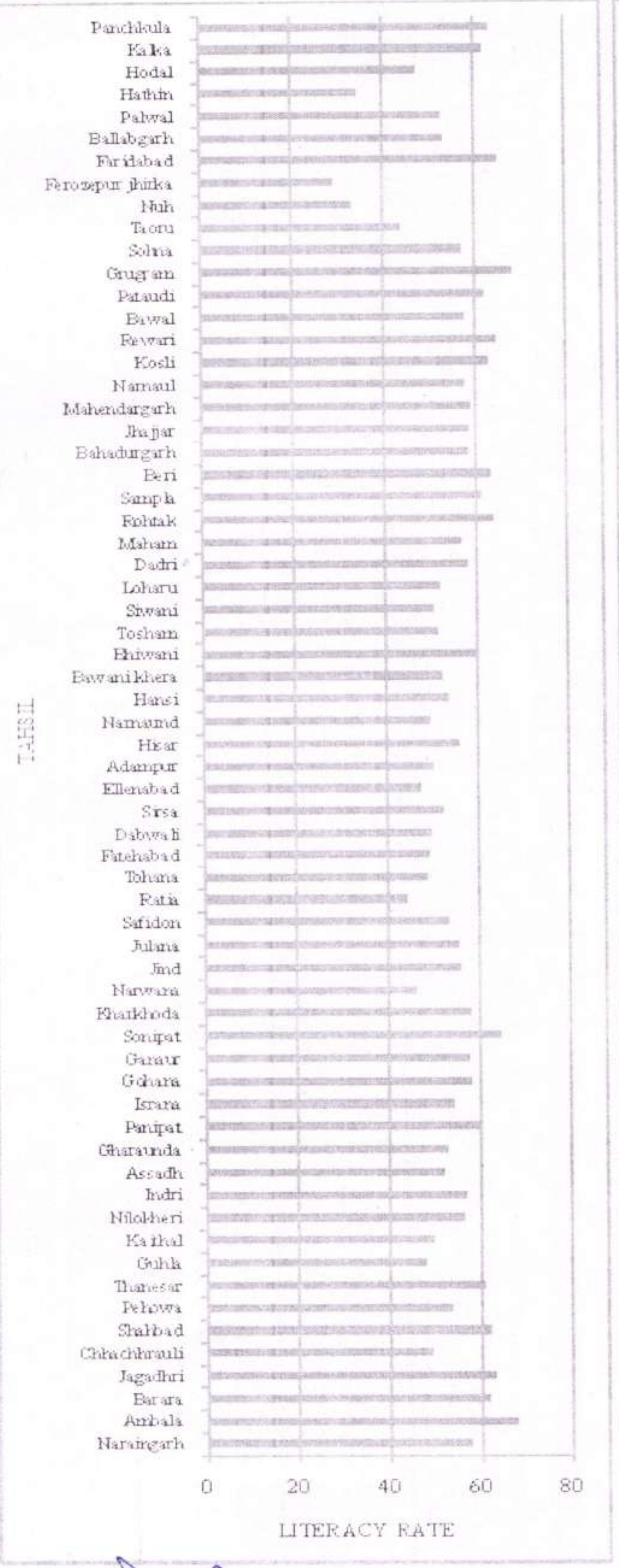
$$\text{Sex Ratio} = \frac{\text{No of females}}{\text{No of Males}} \times 1000$$

Sex ratio, being the second important indicator of human development index; have a worst condition in Haryans in 2011 and 2001 (in India) sex ratio was 940 females per thousand males and 933 females per thousand males respectively. haryana is showing sex ratio below national average in both census years 2011 and 2001. It was 879 and 861 females per thousand males respectively for 2011 and 2001. Haryana is ranked at 31st in case of sex ratio in states and UTs.

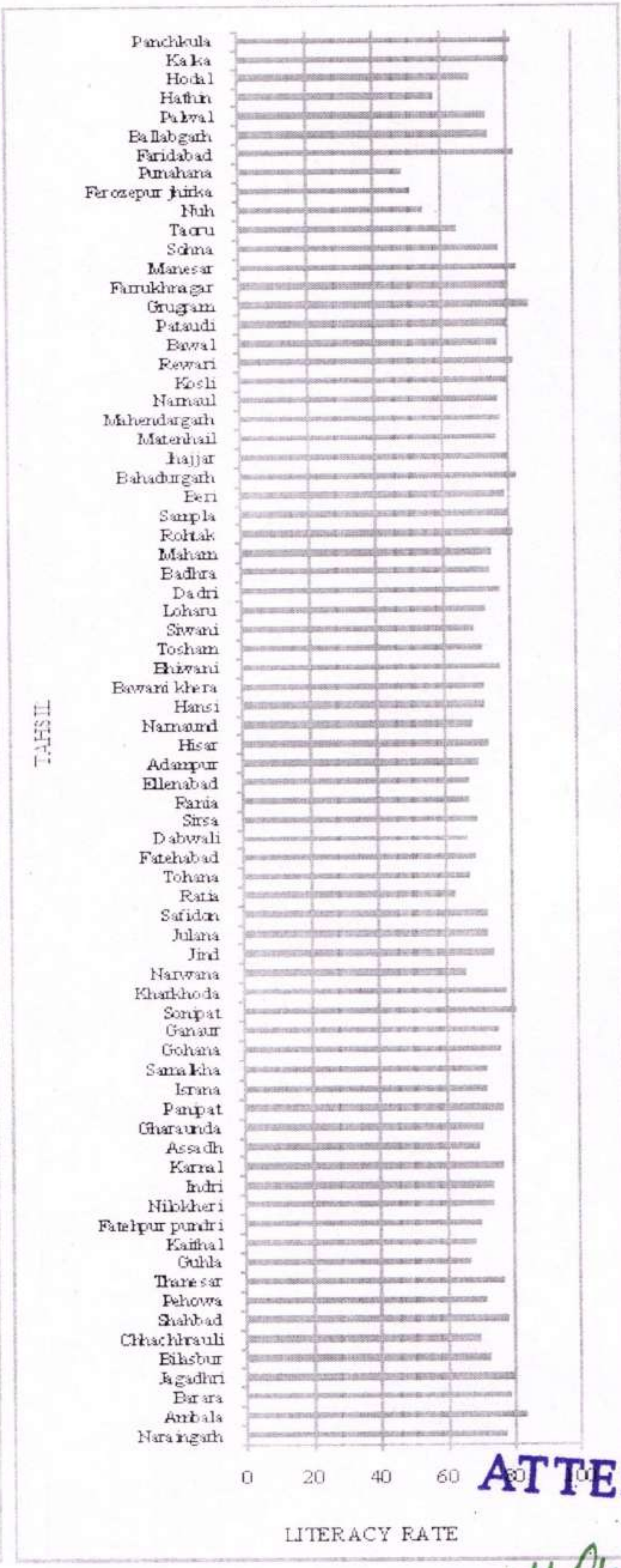
# HARYANA

## TAHSIL WISE LITERACY RATE

2001




2011

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## TAHSIL WISE SEX RATIO

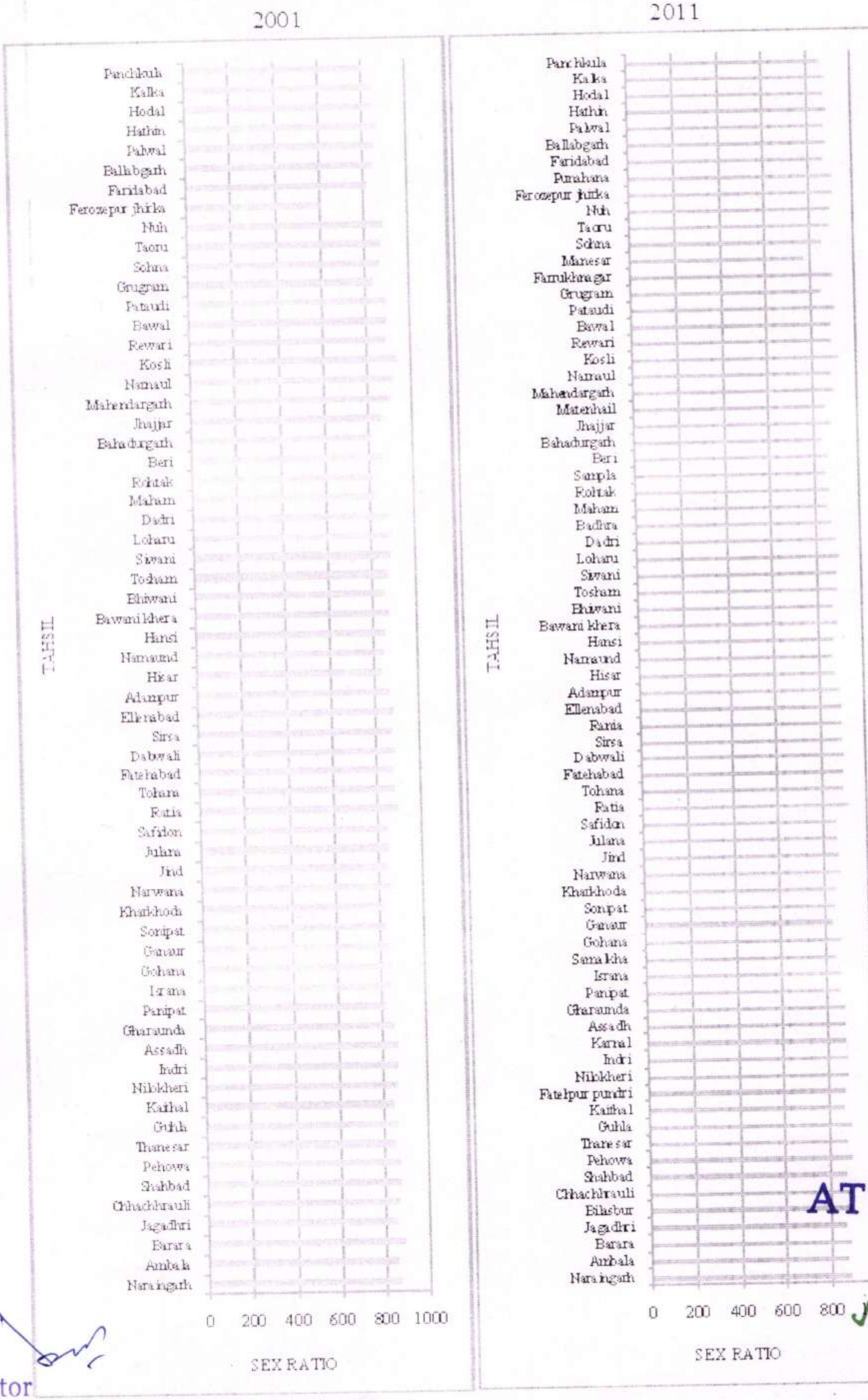


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Table 1.2: Tahsil wise highest and lowest literacy rate and sex ratio for 2001 and 2011.

| S. No. | Literacy Rate | | | | Sex Ratio | | | |
|---|---|---|---|---|---|---|---|---|
| | 2001 | | 2011 | | 2001 | | 2011 | |
| | Highest | Lowest | Highest | Lowest | Highest | Lowest | Highest | Lowest |
| 1 | Grugram (68.80) | Ferozpur Jhirka (29.75) | Hrugram (87.02) | Punahana (49.26) | Kosli (943) | Panchkula (810) | Kosli (925) | Manesar (779) |
| 2 | Ambala (68.33) | Nuh (33.63) | Ambala (84.15) | Ferojpur Jhirka (51.59) | Narnaul (919) | Bahadurgarh (820) | Ratia (918) | Ganaur (840) |

Based on map 2

In Haryana the tahsil wise literacy rate for 2001 and 2011 is shown in fig. 1.1. This depicts that during 2001 the highest literacy rate was in Ambala tahsil (68.33%) and Gurugram tahsil (68.80%) while two tahsil having lowest literacy rate wewe Nuh (33.63%) and Ferojpur Jhirka (29.75%) in Mewat district.

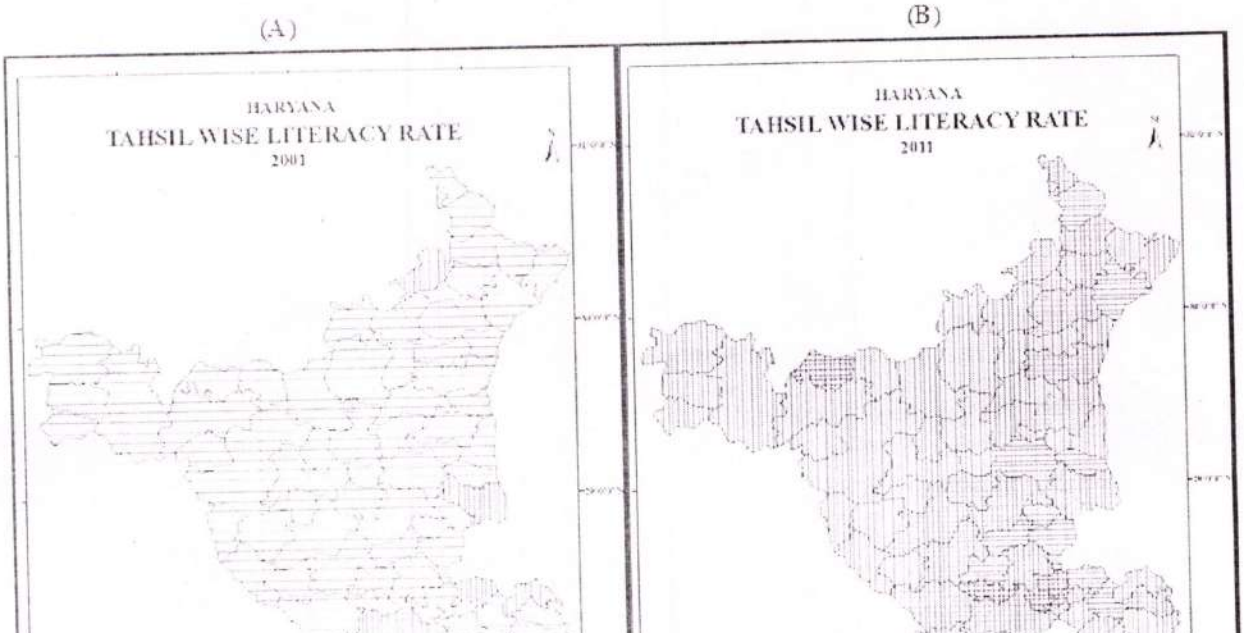
In 2011 the two tahsils having highest literacy rate were same as in census year 2001 i.e. Ambala (84.15%) and Gurugram (87.02%). The literacy rate in 2001 were below 70% ; while it crosses 80% during 2011. The two tahsil having lowest literacy rate were in Ferojpur Jhirka ( 51.59) and Panchkula (49.26) in Mewat district. The analysis shows that the lowest literacy rate exists in Mewat district. The situation is somehow literacy rate.

For a proper understanding of ground level reality; the study of sex ratio at tahsil level is important. For this study tahsil of 2001 and 2011 are considered.

Considering the sex ratio tahsilwise in Haryana; two tahsils having higest sex ratio in 2001 are and lowest sex ratio in Bahadurgarh (820) in Jhajjar district and Panchkula (810). During 2011 the tahsilwis sex ratio was showing average negative change i.e. the thasils with higest sex ratio were Kosli (925) district Rewari, Ratia (918) Fatehabad district and the lowest sex ratio in Manesar (779) in Grugram district and ganaur (840) in Sonipat District.

Map - 2

(A) (B)

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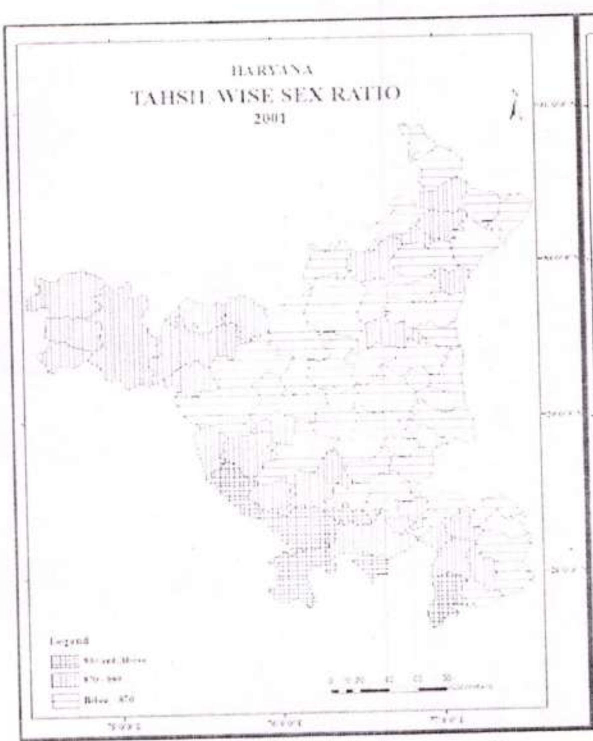


(C)


(D)

Table 1.3: Tahsils wise categorization of literacy rate in 2001 and 2011.

| Literacy Rate | Years | Tahsils |
|---|---|---|
| High Literacy Rate (75 and Above) | 2001 | Nil |
| | 2011 | Panchkula,Kalka,Ballabgarh,Faridabad,Sohna,Manesar,Farrukhnagar,Grugram,Pataudi,Bawal,Rewari,Kosli,Narnaul,Mahendergarh,Matenhail,Jhajjar,Bahadurgarh,Beri,Sampla,Rohtak,Maham,Dadri,Naraingarh,Ambala,Barara,Jagadhri,Shahbad,Thanesar,Karnal,Panipat,Gohana,Ganaur,Sonipat,Kharkhoda,Bhiwani |
| Moderate Literacy Rate (65 - 75) | 2001 | Ambala,Sonipat,Faridabad,Grugram,rewari |
| | 2011 | Hodal,Palwal,Taorn,Bilaspur,Chhachhrauli,Pehawa,Guhla,Kaithal,Fatehpur,Pundri,Nilokheri,Indri,Assandh,Gharaunda,Israna,samalkha,Narwana,Jind,Julana,Safidon,Tohana,Fatehabad,Dabwali,sirsa,Rania,Ellenabad,Adampur,Hisar,Narnaund,Hansi,Bawanikhera,Tosham,Siwani,Loharu |
| Low Literacy Rate (Below - 65) | 2001 | Naraingarh,Barara,Jagadhri,chhachhrauli,Shahbad,Pehowa,Thanesar,Gohana,Guhla,Kaithal,Nilokheri,Indri,Assadh,Gharaunda,Panipat,israna,Ganaur,kharkhoda,Narwana,jind,Julana,Safidon,Ratia,Tohana,Fatehabad,Dabwali,Sirsa,Ellenabad,Adampur,Hisar,Narnaund,Hansi,Bawanikhera,Bhiwani,Tosham,Siwani,Loharu,Panchkula,Kalka,Hodal,Hathin,Palwal,Ballabgarh,Kosli,Narnaul,Mahendargarh,Jhajjar,Ferozpurjhirka,Nuh,Taoru,Sohna,Pataudi,Bawal,Bahadurgarh,Beri,Sampla,Rohtak,Maham,Dadri |
| | 2011 | .Hatin,Punhana,Ferozepur Jhirka,Nuh,Ratia |

Based on map 2 (A) and 2 (B)

Table 1.4: Tahsils wise categorization of sex ratio in 2001 and 2011

| Sex Ratio | Years | Tahsils |
|---|---|---|
| High Sex Ratio (900 and Above) | 2001 | Loharu,Mahendergarh,Narnaul,Kosli,Bawal,Ferojpur Jhirka |
| | 2011 | Chhachhrauli,Pehowa,Guhla,Dabwali,Loharu,Kosli,Patudi,Farruknagar,Ferojpurjhirka,Punahana,Ratia |
| Moderate Sex Ratio (870 -890) | 2001 | Narayangarh,barara,Shahbad,Pehowa,Indri,Assandh,Ratia,Tohana,Fatehabad,Dabwali,Sirsa,Ellenabad,Adampur,Bawanikhera,Tosham,Siwani,Dadri,Rewari,Pataudi,Sohna,Taoru,Nuh,Hathin |
| | 2011 | Palwal,kalka,Taoru,Naraingarh,Ambaka,Barara,Bilaspur,Shahbad,Tohana,Fatehabad,sirsa,Rania,Ellenabad,Adampur,Hisar,Bawanikhera,Tosham,Siwani,Dadri,Badhra,Rohtak,Jhajjar,Mahendargarh,Narnaul,Rewari,Bawal,Nuh,Faridabad,Ballabhgarh |
| Low Sex Ratio (Below - 870) | 2001 | Kalka,Panchkila,Ambala,Jagadhri,Chhachrauli,Thanessar,guhla,Kaithal,Nilokheri,Gharaunda,Panipat,Israna,Gohana,Ganaur,Sonipat,Kharkhoda,Narwana,Jind,Julana,Hisar,bhiwani,Meham,Rohtak,Beri,bahadurgarh,jhajjar,Gurugram,Faridabad,Ballabgarh,Palwal,Hodal, |
| | 2011 | Jagadhri,Panipat,Israna,Sumalkha,Gohana,Ganaur,Sonipat,Kharkhoda,Jind,Julana,Safidon,Narnaund,Hansi,Bhiwani,Maham,Sampla,Beri,Bahadurgarh,Matenhail,Gurugram,Manesar,Sohna,Panchkula |

Based on map - 2 (C) and (D)

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Table 1.3 expresses a great change in the categories for literacy rate and sex ratio for 2001 and 2011. The categorization is shown in fig 2(A) and 2 (B) and discussed as below. **Low literacy rate (below 65%)** : - In this the tahsils håving literacy rate below 65% are cnsidered. In 2001 there were 59 tahsils out of 64; while in 2011 there were only 5 tahsils out of 74. Means there is a great transformation from 2001 to 2011 and the literacy rate has increased at fast speed. **Moderate literacy rate**:- This category represents the literacy rate between 65% to 75%. In 2001 there were only 5 tahsils out of 64 while in 2011 there were 34 tahsils out of 74. **High literacy rate**:- This category consists of literacy rate above 75%. In 2001 there was no tahsil in tis category while in 2022 there were 35 tahsils. Hence the literacy rate shows a drastic change from 2001 to 2011. There is a great positive change in 2011.

The categorization of table 1.4 also expresses the changing scenario of sex ratio from 2001 to 2011. As in fig 2(C) and 2 (D). This is explained as below. **Low sex ratio**:- This category consists those tahsils which have sex ratio below 870. In 2001 there were 31 tahsils while in 2011 there were 23 tahsils. **Moderate sex ratio**:- In this category the range of sex ratio is between 870 to 890 females per 1000 males. During 2001 there were 23 tahsils while there were 29 tahsils in 2011 in this category of moderate sex ratio. **High sex ratio**:- This category represents those tahsils which have sex ratio more than 900 females per 1000 males. During 2001, there were 6 tahsils only while in 2011 there were 11 tahsils. The above categorisation shows a moderate change in sex ratio.

**Spear Man's Rant Correlation**:- For the proper analysis of correlation between sex ratio and literacy rate for 2001 and 2011, Spearman's rank correlation is calculated for all the tahsils. The rank correlation will explain the positive or negative correlation of both. For the census year 2001 the Spearman's rank correlation was (+) 0.23 while the value of rank correlation for 2011 is (-) 0.41. Which explains that sex ratio and literacy rate were positively correlated in 2001, however it becomes a negative low level correlation in 2011. It means that in case of Haryana with increase in literacy rate, sex ratio is decreasing.

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# Struggle for Survival and Dignity: A Study of Maya Angelou's *I Know Why the Caged Bird Sings* and *Gather Together in my Name*

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Abstract

The present paper has explored how racial, class and gender oppression affect the experience of a black woman in the United States of America through the two works of Maya Angelou, an African-American writer. Through *I Know Why the Caged Bird Sings* (1969), Angelou reflects upon how a young woman becomes a victim of racist and sexist ideology in her tender years developing a diminished self-concept leading to identity crisis. *In Gather Together in my Name* (1974), Angelou presents a teenage mother, an unprofessional and unskilled black girl to whom only the most menial jobs are available. For her survival, she tries her hand at all sorts of jobs. Maya's work experience shows how the socially, economically and politically oppressive climate of the United States excludes this woman from the constructive economic engagement and its impact on her as an individual. But what is more commendable on the part of this woman is how she survives all sorts of odds by displaying profound strength and tenacity and emerges as a self-reliant and an independent person.

Keywords: Afro-American Literature, Racism, Sexism, Womanhood, Survival.

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**Introduction**

Maya Angelou's contribution to the literary tradition in America remains unsurpassed. She reconstructs the story of her past. Her first work *I Know Why the Caged Bird Sings* (1969) presents Maya (the main character) a black girl, leading a segregated life in Stamps in the early years of her life. In this town, there was a clear cut division on the racial lines. Angelou avers, "In Stamps, the segregation was so complete that most black children didn't really, absolutely know what whites looked like" (25). This work gives an account of her formative years spent in Stamps, Arkansas and describes her bitter sweet experiences of rural Arkansas, until the birth of a son when she was seventeen fighting against all odds. For a young black girl in the late 1930s and early 1940s, growing up in the South of the United States was a horrifying experience. She seriously deliberates upon the issue of gender as well. She speaks of the countless disturbing experiences and the ugliness of white prejudice thus "living inside a skin that was hated or feared by the majority of one's fellow citizens or about the sensation of getting on a bus on a lovely morning, feeling happy and suddenly seeing the passengers curl their lips in distaste or avert their eyes in revulsion" (*Singin' and Swingin' and Gettin' Merry Like Christmas* 260). Through her works she doesn't articulate her individual experiences but the concerns of the collective.

The Great Depression (1929-1942) stirred the whole of United States. This economic crisis hit the small towns like Stamps as well. There were not many jobs in Stamps at that time. African-American men earned their living by farming whereas some women worked on the cotton plantation farms and others took in washing and ironing in the houses of the white people. As a child she witnessed the abuse faced by the members of her community who in spite of working hard in fields, picking cotton, could never get ahead in life nor come out of their debilitating financial positions. The afternoons, in cotton picking time, present the real harshness of Black Southern life as "in the dying sunlight the people dragged, rather than their empty cotton sacks" (*I Know Why the Caged Bird Sings* 8). All this gives Maya a first-hand knowledge of the condition of Blacks. Here, this perceptive and sensitive child Maya and people of her community experience closely America's troubled legacy of racism, intolerance, violence and cultural divide against the African-American community. Blacks face many other challenges while living in a White dominated society. Angelou's autobiographies vivify the Southern life as a trouble spot with harshness and brutality incurred by the whites. White dominance opens the eyes of Maya to the harsh reality that she is the member of an oppressed and deprived class. Through her first work Angelou demonstrates the life of South as full of violence and despair. Growing up in Arkansas, living under the terror of Ku Klux Klan - a white supremacist group, unwarranted ...

racism was rampant, a black person could remain safe only when he stayed away from the white people. The cruelty of whites makes the existence of blacks difficult in the society.

Angelou recounts many such episodes where she comes across racist characters who try to victimize her and members of her race in one way or the other. She recalls an incident of her childhood when she was just ten years old but later on it proved to be a pivotal experience of her life for it taught her how to survive and live with dignity even in unfavourable conditions. This episode involves three 'powhitetrash' girls who visit their store. They taunt Maya's grandmother Mrs. Henderson (Momma), imitate her posture and her mannerism. Leaving aside all decency, they even address her insolently by her first name. But Mrs. Henderson stands undisturbed like a rock, humming a hymn and smiling throughout the scene. When Mrs. Henderson does not give in, they devise other ways of irritating, imitating her, doing handstands and calling dirty names. Young Maya, feeling sorry for her grandmother, wants to retaliate and teach them a good lesson by confronting them literally. But soon Maya realizes that she is "as clearly imprisoned behind the scene as the actors outside are confined to their roles" (30). Grandmother taught Maya "how to act around Whites without losing their dignity" (Cox 3). While interpreting this episode, Dolly McPherson in *Order Out of Chaos* (1990) finds the confrontation as an example of Powhitetrash girls using their "power to treat Black woman like another child" (32). In this scene the black woman and her granddaughter adopt the dignified course of silent endurance. This scene recaptures the black/white tension in the South of United States in 1930s. Whites have full sanction of White community to practice power to belittle blacks which can be noted on other occasions as well.

Angelou gives many instances to vivify the precarious condition of the blacks. Maya notices how vulnerable his Uncle Willie is in spite of being crippled. His lameness offers him no protection and he too has to hide in the potato bin to protect himself from the wrath of Ku Klux Klan. Other incidents also provide a proof of a ritualistic violence of the White world against Blacks. Maya's brother Bailey gets horrified when a local white asked this just fourteen-years old to help him in disposing-off the body of a dead and rotten black male. On another occasion, he started asking questions when he happens to see the emasculated body of another black man. This incident makes Maya reach the conclusion that "the Black woman in the South who raises sons, grandsons and nephews had her heartstrings tied to a hanging noose" (*I Know Why the Caged Bird Sings* 114).

The description of the Graduation Ceremony in Maya's school is one such episode. Maya and her brother Bailey study in Lafayette County Training School meant only for the black children. Marguerite remembers her school which "distinguished itself by having

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neither lawn, nor hedges, nor tennis court . . . its two buildings . . . were set on a dirt hill . . ." (170). Education means much to the blacks. They believe that only through education a social change in the white dominated society is possible. Their boys and girls can come out of white man's kitchen. They see in higher education the possibility of personal and racial liberation. People engaged in the black liberation struggle emphasized the importance of education for this very reason. That is why the tradition among Negroes to give present to the children going from one grade to another was much popular. The Graduation day was so important to them that "parents who could afford it had ordered new shoes and ready-made clothes for themselves . . . which would be pressed to a military slickness for the important event" (171). Graduation Ceremony, a momentous occasion for the graduates as well as the whole community, is marred when a white guest speaker, Mr. Donleavy, from Texarkana, tells the graduating class about the possibility of career opportunities available to young black men and women. Promising the white students a future full of advanced educational opportunities and praising the black community for producing good sportsmen, he shatters the hopes of blacks gathered there to have a bright future. Angelou's racial awareness grows and she puts forth her thoughts into words to share with everybody be it white or black audience: "the white kids were going to have a chance to become Galileos and madam Curies and Edisons and Gaugins, and our boys (the girls weren't even in on it) would try to be Jesse Owenses and Joe Lousies" (179). She further remarks, "We were maids and farmers, handymen and washerwomen, and anything higher that we aspired to was farcical and presumptuous" (180). At this stage the pain is evident in Angelou's words, "It was awful to be Negro and have no control over my life. It was brutal to be young and already trained to sit quietly and listen to charges brought against my colour with no chance of defence. We should all be dead" (180). By extolling the whites and demeaning the blacks this guest speaker turns the sunshine into a cloud of ugliness by delivering a racist speech meant especially for this occasion. He succeeds in dampening the spirit of black students to some extent for some time. Maya loses her sense of identity after Mr. Donleavy's address. She feels, "My name had lost its ring of familiarity and I had to be nudged to go and receive my diploma" (180). The white society negates her identity.

In her movement from childhood to adolescence, that is, from innocence to awareness, Angelou records certain social barriers she confronts and tries to overcome in order to assert a sense of self and relative freedom. Her first experience with a white person catapults her into the realization of her social reality and into a growing consciousness of self-worth. The encounter with Mrs. Viola Cullinan, a wealthy, Virginian settled in Stamps for whom Maya works as a domestic help proves to be a major turning point. With the pretensions of a Southern white woman, this woman has no respect for the individ name that Mrs. Cullinan finds too complicated to pronounce. It was the tradition of White Americans to change a black person's name for convenience. A name is generally considered the symbol of one's identity. By mispronouncing Maya's name, Mrs. Cullinan rejects her humanity. Maya who is sensitive, intelligent and alert comprehends the nature of this insult. Mrs. Cullinan's episode in *I Know Why the Caged Bird Sings* is quite salient as it points towards the significance of the realm of the 'personal'. This leads Marguerite to seek revenge on her by breaking some of her treasured heirlooms thus asserting herself as an individual. Maya Angelou aptly remarks:

> Every person I knew had a hellish horror of being "called out of his name." It was a dangerous practice to call a negro anything that could be loosely constructed as insulting because of the centuries of their having been called niggers, jigs, dinges, blackbirds, crows, boots and spooks. (109)

The episode in which Maya and her grandmother go to a dentist for Maya's treatment and the dentist refuses to put hand in Maya's mouth saying "I'd rather stick my hand in a dog's mouth than in a nigger's"(189). The way derogatory word like "nigger" is used for a black person points towards the clashes between whites and blacks and whites attitude towards blacks. African-American literature is abounding with this theme of dehumanization where a black person is seen as devoid of basic human attributes. The hero of Richard Wright's *The Man Who Lived Underground* (1944) behaves like a dog. *If We Must Die* (1919), a poem by Claude Mc Kay, uses a series of animal references to convey the brutal attitude of whites towards blacks in America as hogs, barking dogs, and pack of dog-like men.

If seen historically, the black women in America have been doubly oppressed, caught in the slugfest of racist and sexist groups. They always feel ashamed of their black colour as if it is an albatross hanging around their neck. W.E.B. Du Bois in *The Souls of Black Folk* (1903) has rightly observed: "The problem of the twentieth century is the problem of the colour-line - the relation of the darker to the lighter races of men in Asia and Africa, in America and the islands of the sea" (41). American society accorded white colour of skin as a positive attribute and a pre-requisite for success. Possibly no other social group has ever become a victim of such an unedifying spectacle of human debasement and moral depravity. As a result black women developed the neurosis of self-hatred and self-censorship, both individually and as a part of a group. They have made constant efforts (though largely in vein) to assimilate themselves in the white culture through internalizing white beauty ideals. It is a peculiar sensation which can be called a "double consciousness", a phrase devised by Du Bois which means "always looking at one's self through the eyes of others, ... one's soul by the tape of a world that looks on in amused contempt and pity" ... results in self-hatred. Calvin Hernton in *Sex and Racism in America* (1965) makes the following observation:

> The attempt to become white intensifies rather than mitigates the Negro woman's frustration in White world. No amount of paint, powder and hair straightener can erase all the things in the black woman's background that make her femininity and aesthetic appreciation of herself as a beauty capable of attracting men. The Negro woman feels ashamed of what she is. (133)

The society in which young Maya lives not only defines beauty in terms of standards set by white people, but also makes one internalize these notions. Afro-American writers talk about many such black women characters in their works who pursue the white beauty ideals. Pecola Breedlove in Toni Morrison's *The Bluest Eye*, Maud Martha in Gwendolyn Brook's *Maud Martha*, Selina Boyce in Paul Marshall's Brown Girl, Florence in James Baldwin's *Go Tell it on the Mountain*, Marguerite in Maya Angelou's *I Know Why the Caged Bird Sings* are among some of them who fly on the wings of white fantasy. Marguerite fantasizes:

> Wouldn't they be surprised when one day I woke out of my black ugly dream, and my real hair, which was long and blond, would take the place of the kinky mass that Momma wouldn't let me straighten? My light blue eyes were going to hypnotize them. (*I Know Why the Caged Bird Sings* 2)

Later in conversation with her friend Rosa Guy, Angelou observes, "My belief as a child that I was ugly was absolute, and nobody tried to disabuse me--not even Momma. Momma's love enfolded me like an umbrella but at no time did she try to dissuade me of my belief that I was an ugly child" (Elliot 235). This letter shows the repercussions of social conditioning on a sensitive child growing up in an uncongenial environment. Black psychiatrists William H. Grier and Price M. Cobb in their seminal work *Black Rage* (1992), through various case studies, reflect upon the psychological effect of perpetual confrontation between a debased self-image of a black girl/woman and an elevated self-image of a white woman can have on the psyche of a black girl/woman:

> Her blackness is the antithesis of a creamy white skin, her lips are thick, her hair is kinky and short. She is, in fact, the antithesis of American beauty. However beautiful she might be in a different setting with different standards, in this country she is ugly... There can be no doubt that she will develop a damaged self-concept and an impairment of her feminine narcissism which will have profound consequences for her character development. (41)

The non-realization of these internalized dreams generally leads the black women to self-deprecation, undermining themselves. This sense of loss of self-worth leads to self-... and low self-esteem which ultimately makes the person develo...

and a fragmented image. Sidonie Ann Smith in the article, "*The Song of a Caged Bird*," comments:

> Maya Angelou's autobiography... opens with a primal childhood scene that brings into focus the nature of the imprisoning environment from which the self will escape. The black girl child is trapped within the cage of her own diminished self-image around which interlock the bars of natural and social forces. (6)

The way Angelou envisions herself in her childhood days, fighting for identity, epitomizes the plight and trial of the black community as such. In *Black Rage* (1992) psychiatrists William H. Grier and Price M. Cobb describe this "imprisonment":

> If the society says that to be attractive is to be white, the Black woman finds herself unwittingly striving to be something she cannot possibly be; and if femininity is rooted in feeling oneself eminently lovable, then a society which views her as unattractive and repellent has also denied her this fundamental wellspring of femininity. (49)

Racial prejudice and economic depression can be held responsible for this diminished sense of self of an individual as well as the entire black community. Angelou's Maya also becomes a victim of this 'identity crisis' as she runs after ideals of white feminine beauty. She too becomes victim of ideologies which propagate and accept white colour as the mark of real beauty. Afro-American literature is abound with such women characters who testify to the black women's quest for white beauty ideals. Having internalized these false ideals, the black woman fully ignores the psychological aspect of her true self. She isolates herself from the 'whys' and 'hows' of her own existential conditions. This quest makes them the culturally alienated pariahs. Pathetically divorced from their own cultural system, the black woman runs after the white bourgeois ideals and finally land into the quagmire of delusion. The person who denounces his own cultural or racial self and adopts other's mandates always remains in a suspended position. The culture which would accept him he rejects and the culture which rejects him, he accepts. Thus rejected by both the cultures he remains in a state of cultural limbo. The person who internalizes alien cultural value system becomes a victim of darkness within his own psyche. Instead of asserting, such a person sometimes chooses not to say anything and in a way support his own oppression.

Through this work Angelou demonstrates the manner in which any black female is violated by "masculine prejudice, white illogical hate and black lack of power" (*I Know Why the Caged Bird Sings* 231) in her tender years and it also demonstrates the "unnecessary insult" every southern girl faces in her growth to adolescence. After living several years in Stamps, Arkansas and even after suffering all sorts of insults and shame delivered by White people from all walks of life, Maya has been taught by the men...

her community and extended family to feel proud to be the member of this race and assert her identity.

*Gather Together in my Name* (1974) is the second Angelou's work. Maya the young child has become a mother now and her struggle to survive as a black woman in white America has been portrayed in this work. Angelou states the conflict Maya faces at the beginning: "I was seventeen, very old, embarrassingly young, with a son of two months, and I still lived with my mother and stepfather" (5). Maya's character passes through this ambivalence where she finds herself too old and too young simultaneously, too responsible (she is a mother) but too dependent. The place is San Francisco and the time is just after the end of World War II, mid 1940s leaving soldiers hang around "the ghetto corners like forgotten laundry left on a backyard fence" (5). The illusion of racial equality, with which Blacks migrated to North in search of better job opportunities and less racial discrimination, begins to vanish after World War II.

The job market crashes and flow of easy money stops. Maya, now seventeen with the responsibility of a child looks for a job but unfortunately she is a black girl who is bound to face discrimination because of her colour. This is how Maya begins her journey where in order to support her son and herself she takes up all sorts of menial jobs. She serves as a short order cook, a nightclub dancer and a waitress. She runs own house of prostitution. This long list of menial jobs (which includes even those on the fringes of society) that she pursues do not ensure her financial security. In this post war milieu, evil abounds. She comes in contact with pimps, drug addicts, con men and street women, gamblers, black-marketeers, boosters and a lover who steals for his living. At one point in the book she herself works as a prostitute to help her perfidious boyfriend, to whom she is planning to marry, her "sugardaddy" (a pimp known by this name) from debt. This situation demonstrates how black women are abused not only by whites but also by blacks. Black men try to victimize their women in every conceivable way. They value their women only because of their bodies. Their bodies provide emotional strength to them when they are lost, lonely or bewildered. Calvin Hernton in his work *The Sexual Mountain and Black Women Writers: Adventures in Sex, Literature and Real Life* (1990) observes succinctly: "Although black and white men stand on opposite sides of racial mountain in America they tread on common ground when it comes to the mountain of sex" (82).

In such a situation where a young black mother wrestles with the need to provide for her baby and is herself quite vulnerable, she is bound to make easy choices keeping aside the moral values learnt in the South. The alienation and fragmentation of the urban north overpowers dignified and ethical manners of the rural South. These are the conditions that Maya encounters when she tries to situate herself and struggles to survive in a ...

The stage of life that Maya Angelou depicts in *Gather Together in My Name* is full of uncertainties. She does not know who she is or what role is most conducive and appropriate for her. Her quest for her place in the scheme of things, as a black woman in America, still bothers her. In this state of restlessness, frustration, trying-on of roles, Maya undergoes various experiences and through self-education moves from innocence towards maturity and adulthood. This work presents the conditions in which the unemployed, drug addict, blacks and others become criminals and the situations in which they are bound to live, "Most of the friends, funny and bright during schooldays... sparkling young men who were hopes of the community had thrown themselves against the sealed doors set up by a larger community" (131) and this scene which presents chaos and destruction, shows a black person's slide into iniquity in a white society. The white society acts villainously and criminally as it reduces all Negro men to nothingness and impotence and women to lead lives of whoredom and destitution.

For Maya this milieu becomes a point where her struggle to restore the sense of dignity and personhood starts which is a necessary prerequisite for expressing any sense of womanhood or racial identity. Maya, with the strength of her mind, manages to survive in this world which is otherwise full of filth, without dignity and purpose especially for a black woman. The themes that recur throughout all her works are courage, perseverance, persistent effort against overwhelming obstacles and moving on the path of attaining selfhood in spite of all hindrances and the last but not the least 'Survival'. Angelou in one of her works, *A Song Flung up to Heaven* (2002), poignantly remarks:

> How did it happen that we could nurse a nation of strangers, be maids to multitudes of people who scorned us, and still walk with some majesty and stand with a degree of pride? I thought of human beings, as far back as I had read, of our deeds and didoes. According to some scientists, we were born to forever crawl in swamps, but for some not yet explained reason, we decided to stand erect, and despite gravity's pull and push, to remain standing. (211)

African-American women learnt to assert themselves by protesting against this discrimination. They struggled for equal wages for equal work, better working conditions in laundry, garment, service industry. Instead of remaining passive victims of oppression they became pillars of strength for their families and worked incessantly and remarkably for the upliftment of their race. Black feminist critic Patricia Hills Collins in *Black Feminist Thought: Knowledge, Consciousness and the Politics of Empowerment* (1990) says, "the voices of these African-American women are not those of victims but of survivors" (109).

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Angelou too realises that a self-defined and well-articulated woman's voice is the first step towards women's survival. Her greatest achievement lies in her ability to transcend her personal pain and stand up for what is right. Her stories of courage and perseverance encourage others also. Blackness and womanness were things beyond control, yet she strove to break these stereotypes to some extent and emerged victorious. She survives all sorts of odds by displaying profound strength and tenacity. In such a situation a weak willed person would have plunged into depression but young Maya falls back on her diligence, her inner strength and emerge as a self-reliant and independent African American woman, emboldened by her long suffering and oppression.

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